Essays on testing for spanning and on modeling futures risk premia

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Essays on Testing for Spanning and on Modeling Futures Risk Premia

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Essays on Testing for Spanning and on Modeling Futures Risk Premia

Frans de Roon

Katholieke Universiteit Brabant, december 1997

I

De voordelen van het beleggen in de zogenaamde opkomende markten kunnen worden geïllustreerd door te laten zien dat de efficiënte grenslijn van beleggingen in markten zoals die van de V.S., Europa en Japan aanzienlijk verschuift wanneer hieraan beleggingen in de opkomende markten worden toegevoegd. Betrouwbaardere uitspraken omtrent deze voordelen kunnen worden gedaan door een statistische toets uit te voeren of de efficiënte grenslijn inderdaad verschuift wanneer beleggingen in opkomende markten worden toegevoegd. Wanneer bij een dergelijke toets rekening wordt gehouden met het feit dat short selling niet is toegestaan en met het bestaan van transactiekosten, dan blijkt dat er slechts weinig bewijs is voor de voordelen van het beleggen in opkomende markten. [Hoofdstuk 4 van dit proefschrift]

II

De prestatiemeting (performance evaluation) van beleggingsfondsen, futures contracten, beleggingen in opkomende markten en andere beleggingsvormen, kan worden gedaan met behulp van de (gegeneraliseerde) Jensen maatstaf, die verkregen wordt als het intercept in een regressie van de zogenaamde excess-rendementen van de te evalueren beleggingsvorm op een constante en de excess-rendementen van de benchmark-portefeuilles. Een belegger die uitsluitend geïnteresseerd is in het verwachte rendement op zijn portefeuille en de variantie van zijn portefeuillerendement, kan met behulp van de geschatte regressieparameters bepalen hoeveel hij van de nieuwe beleggingsvorm moet (ver)kopen en hoeveel hij van zijn benchmark-portefeuilles moet bij- of verkopen teneinde een optimale portefeuille te verkrijgen. [Hoofdstuk 2 van dit proefschrift]

III

In tegenstelling tot de gangbare publieke opinie leren de economische theorie en de empirie dat in financiële markten speculanten een nuttige functie kunnen vervullen. [Hoofdstuk 6 van dit proefschrift]

IV

Het feit dat termijn- en optiemarkten gekarakteriseerd kunnen worden als zero-sum games, wil niet zeggen dat deze markten op casino's lijken. Daar waar in casino's twee partijen met elkaar in contact treden om hun beider risico te vergroten, kunnen twee partijen op eerder genoemde markten met elkaar in contact treden om hun beider risico juist te reduceren. [Hoofdstuk 5 en 6 van dit proefschrift]

V

De vruchtbare interactie tussen het vakgebied financiering en het vakgebied econometrie is niet verwonderlijk gegeven het feit dat één van de centrale problemen in beide vakgebieden het minimaliseren van (residuele) variantie is (al dan niet onder nevenvoorwaarden). [Hoofdstuk 2 van dit proefschrift]

VI

Het econom(etr)ie-onderwijs aan Nederlandse universiteiten, waarin studenten reeds in een vroeg stadium kiezen voor een studie (bedrijfs)economie danwel econometrie, leidt ertoe dat studenten (bedrijfs)economie te weinig kennis hebben van econometrie en omgekeerd. Dit wreekt zich met name in de financiële economie.

VII

Het gedrag van beleggers die hun aan- en verkoopbeslissingen baseren op recente koersbewegingen is zeer wel vergelijkbaar met het gedrag van automobilisten die tijdens het filerijden steeds van rijbaan wisselen.

VIII

De invoering van een gezamenlijke Europese munt geeft blijk van de menselijke neiging om eerst makkelijke problemen op te lossen in plaats van relevante.

IX

Door technologische ontwikkeling en steeds verder gaande specialisatie worden mensen steeds afhankelijker van elkaar. Een logische en natuurlijke reactie hierop is wat genoemd wordt de individualisering van de samenleving.

X

Een steeds groter deel van het sociale leven van promovendi speelt zich af via e-mail.

XI

Het nadenken over de vraag wat de zin van het leven is, is één manier om zin te geven aan het leven. Het lijkt echter niet zinnig te verwachten een bevredigend antwoord op deze vraag te vinden.

XII

Zowel voor werkende alleenstaanden als voor tweeverdieners is de avondopenstelling van supermarkten een zegen.

Essays on Testing for Spanning and on Modeling Futures Risk Premia

Essays on Testing for Spanning and on Modeling Futures Risk Premia

Proefschrift

ter verkrijging van de graad van doctor aan de Katholieke Universiteit Brabant, op gezag van de rector magnificus, prof. dr. L.F.W. de Klerk, in het openbaar te verdedigen ten overstaan van een door het college van decanen aangewezen commissie in de aula van de Universiteit op woensdag 17 december 1997 om 14.15 uur door

Frans Adrianus de Roon

geboren op 19 september 1969 te Sprang-Capelle

Promotor: Prof. dr. Th.E. Nijman

Copromotor: Dr. C.H. Veld

Preface

A large part of the research reported in this thesis has appeared elsewhere. Chapters 3 and 4 contain the results of DeRoon, Nijman & Werker (1997a) and (1997b) respectively. Chapter 6 is a slightly revised version of DeRoon, Nijman & Veld (1997a). Finally, Chapter 7 is virtually identical to DeRoon, Nijman & Veld (1997b) and is forthcoming in the *Journal of Financial and Quantitative Analysis*.

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This study presents the results of three years work at the Department of Finance and CentER for Economic Research of Tilburg University and of one year at the Department of Finance of the Erasmus University of Rotterdam. I could never have written this study in its present form without the help and support of a number of people. Three persons in particular deserve special credits.

First and foremost, I want to express my gratitude to my supervisor Theo Nijman. Theo has been a great stimulator and advisor since the very beginning of my Ph.D. project. He has always given me new ideas to work on and has always been open, interested and enthusiastic towards my own ideas and questions, not only concerning research but many other areas - both inside and outside the university - as well.

Second, I want to thank my supervisor Chris Veld, who stimulated me to become a Ph.D. student at Tilburg University at a very early stage and who has involved me in many of his projects and ideas. Besides that, Chris has become a very good friend over the past four years.

Third, I am grateful to Bas Werker who, being a statistician, had the patience to answer all the silly questions a business economist can ask, and who was always available for (exciting) discussions about our joint papers.

Without naming them specifically (which would easily fill half a page) I would like to thank my colleagues at Tilburg University and Erasmus University for providing a pleasant and stimulating working environment. In the same vein, I also want to thank many colleagues and friends from other universities as well as from outside the university for their continuous support and interest - either directly or via e-mail. A special word of thanks goes to (in alphabetical order) Geert Bekaert, Erzo Luttmer, Piet Moerland, Peter Schotman and Ton Vorst, for their willingness to participate in the Ph.D. committee and for the comments and suggestions they made with respect to several chapters of this thesis.

I want to thank Lysanne for simply being there during the past years. Finally, I am grateful to my parents for giving me the freedom to study the things that really interested me and for supporting me in whatever choice I made.

Frans de Roon
Tilburg, October 1997

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Chapter 1

Introduction

1.1 Introduction

The portfolio choices of investors and asset pricing are two important topics in financial economics. These two topics are the main theme of this thesis. As the title already suggests, this thesis consists of two parts. The first part, which is about spanning and intersection, mainly focuses on the portfolio choices of investors. The second part, which is about risk premia in futures markets, analyzes asset pricing models for a specific subset of securities: futures contracts. The analysis in the first part builds on the well known mean-variance portfolio theory of Markowitz (1952) and the volatility bounds introduced by Hansen & Jagannathan (1991), which are two corner stones of modern finance. For a given set of assets, Markowitz' portfolio theory gives the set of optimal portfolios for investors with utility functions that depend on the mean and variance of portfolio returns only. These optimal portfolios define the mean-variance efficient set. In terms of mean-variance analysis, the central question in the first part of this thesis is whether investors can extend their efficient set by including additional securities in their portfolio. This is the type of question that investors face, e.g., when deciding whether or not to invest in international stocks besides their investment in domestic stocks, whether or not to invest in emerging markets, or whether or not to hedge their exposure to commodity and currency risk.

Starting from the observation that rational agents choose their portfolios and consumption to maximize their expected utility, Hansen & Jagannathan (1991) analyze the implications of security returns for investors' intertemporal marginal rates of substitution (IMRS's). Specifically, they derive restrictions on the mean and variance of each investors' IMRS from a given set of asset returns, which results in a mean-variance frontier for IMRS's. This mean-variance frontier of IMRS's will be referred to as the volatility bound, in order to avoid confusion with the familiar Markowitz mean-variance frontier of portfolio returns. There is a duality between Markowitz' mean-variance frontier and the volatility bound for a set of securities, in that each point on the mean-variance frontier corresponds to a unique point on the volatility bound[1]. The question whether including additional securities in investors' portfolios extends their efficient set can

[1] There is one exception to this duality, which will be discussed in Chapter 2 of this thesis.

therefore also be posed in terms of volatility bounds. In that case the question is whether the returns of additional securities impose tighter restrictions on the mean and variance of IMRS's than the restrictions derived from the initial set of securities. This is the type of question that economists face when they confront their asset pricing models with the data.

The second part of this thesis consists of two essays on risk premia in futures markets. There are at least three characteristics of futures markets that affect expected futures returns or futures risk premia. First, a distinctive feature of futures contracts as opposed to assets like stocks and bonds is that futures contracts do not require an initial investment. This feature will already be discussed in the first part of this thesis. Second, futures contracts are often used to hedge a position that an investor can not or does not want to trade. For instance, importing and exporting firms use currency futures to hedge their exposure to foreign currency risk and many farmers and industrial companies use futures to hedge the commodity price risk that is associated with their business. The fact that investors choose their portfolios taking into account their nontradable or nonmarketable positions, affects their aggregate demand for futures contracts (as well as for other assets) and thereby futures risk premia. Therefore, in the second part of this thesis we will analyze a model for futures risk premia in which the effect of nonmarketable assets on the investors portfolio choice is taken into account. Third, it is usually the case that for a specific asset several futures contracts are traded that differ in the delivery date. This means that at each date we can observe a term structure of futures prices for a specific asset. The expected returns on futures contracts that differ in their delivery date only, will in general be different but can be expected to be closely related to each other. This characteristic will also be analyzed in Part II of this thesis, where we will use a simple one-factor model to analyze term structure risk in futures markets.

Although the two parts of this thesis cover different subjects, there are some obvious links between the two parts. On the one hand, the aggregate portfolio choice of all investors determines the aggregate demand for the available securities, and therefore their prices and expected returns. This will be most apparent in Chapter 6, where we derive futures risk premia from the aggregate demand of investors for financial securities, taking into account the presence of nontradable positions. In analyzing futures risk premia Chapter 6 builds on a lot of material that is derived in Part I of the thesis. On the other hand, the models for futures risk premia that are analyzed in Part II, identify a number of variables that can be used to predict futures returns. Obviously, such variables can be used by investors as conditioning variables in determining their optimal portfolio. Therefore, when using conditioning variables in the analysis of Part I, we will build in part on the results of Part II.

1.2 Outline of the thesis

The first part of the thesis consists of three chapters. **Chapter 2** gives an introduction to the concept of mean-variance spanning and intersection. Because most of the results build on mean-variance frontiers and volatility bounds, Chapter 2 starts with an introduction to these topics and also shows the duality between these two frontiers. When the mean-variance frontier and the volatility bound are derived from a given set of assets, adding new assets to this set may result in a shift in the mean-variance frontier and the volatility bound. If there is a shift in the mean-variance frontier, but the old and the new frontier have exactly one point in common, then there is *intersection*. If the old and the new mean-variance frontier coincide, then there is *spanning*. We will show that the duality between mean-variance frontiers and volatility bounds implies that intersection (spanning) of the mean-variance frontiers is equivalent to intersection (spanning) of the volatility bounds. In Chapter 2 we will also show how we can use regression analysis to statistically test for mean-variance spanning and intersection and how mean-variance spanning and intersection is related to other topics in the finance literature, such as tests for mean-variance efficiency, performance evaluation and specification error bounds. Building on the results in Chapter 2, in **Chapter 3** we extend the regression based tests for spanning in three directions. First of all we show how we can extend the tests for spanning to allow for non mean-variance utility functions as well. Second, we show how to test for spanning and intersection in case some of the assets are zero-investment securities such as futures contracts. Finally, it is shown how the presence of nontraded or nonmarketable positions, which we already referred to above, can be incorporated in tests for spanning and intersection. These extensions are illustrated for investors that have a mean-variance utility function, a log utility function, or a power utility function. Assuming that the investors initially invest in three international stock indices and that they may have a nonmarketable position in some commodity or currency, we test whether the mean-variance frontier of the three international stock indices spans the mean-variance frontier of these same indices plus one of a number of futures contracts.

The analysis in Chapters 2 and 3 assumes that there are no market frictions such as short sales constraints and transaction costs. In **Chapter 4** we relax these assumptions and derive regression based tests for spanning and intersection that take short sales constraints and transaction costs into account. These tests are applied to tests for the diversification benefits of emerging markets. There have been several studies in the literature (see, e.g., DeSantis (1994) and Harvey (1995)) that suggest that including emerging markets in a portfolio offers substantial diversification benefits for investors that already hold an internationally diversified portfolio of stocks from well-developed countries. In Chapter 4 we analyze whether the diversification benefits of the emerging markets are statistically significant when short sales constraints and transaction costs are taken into account.

The second part of the thesis also consists of three chapters. **Chapter 5** gives a short introduction to risk premia in futures markets. There we also elaborate on some links between the first part and the second part of the thesis. In particular, we will show how knowledge about futures risk premia can be used in portfolio and hedge decisions.

In **Chapter 6** we derive a model for futures risk premia where investors may have nonmarketable positions which they take into account in choosing their optimal portfolio. The setup of this chapter is therefore closely related to the setup in Chapter 3. When aggregating the demand for securities of all investors, which are assumed to have mean-variance utility functions, it turns out that futures risk premia in the model depend on the covariance with the market portfolio and on the net nonmarketable positions of all investors, which will be denoted as *hedging pressure*. We analyze the effect of hedging pressure variables on futures returns for a set of 20 futures contracts, containing both financial and non-financial futures. In analyzing the model we use the specification error bounds that were recently introduced by Hansen & Jagannathan (1997) and that can be used as a measure for model misspecification. These specification error bounds are also discussed in Chapter 2 in relation to mean-variance intersection.

As noted above, a characteristic of futures markets is that at each date there are typically several futures contracts traded that have the same underlying asset, but that differ in their delivery date. The risk premia on these futures contracts will differ because of term structure risk in futures markets. This term structure risk arises because of risk premia in the cost-of-carry of the asset underlying the futures contract. In **Chapter 7** we will analyze this term structure risk using the regression framework employed for instance by Fama (1984a, 1984b, 1986) and using an affine one-factor model for the cost-of-carry. In this framework we will analyze risk premia in the term structure of gold, heating oil, live cattle, soybeans and German mark futures.

Finally, **Chapter 8** summarizes the main results in this thesis.

PART I

Testing for Spanning and Intersection

Chapter 2

An Introduction to Mean-Variance Spanning, Intersection, and Related Topics

2.1 Introduction

In this chapter we will provide an introduction to the concept of mean-variance spanning and intersection, as well as to its relationships with volatility bounds, tests for mean-variance efficiency, performance evaluation and the specification error bounds that have recently been proposed by Hansen & Jagannathan (1997). There exists a vast literature on most of these subjects and the intention here is not to give a complete overview, but merely to illustrate that the concept of mean-variance spanning and intersection provides a framework in which many other results can be understood. The main purpose in the literature on mean-variance spanning and intersection is to study the effect that the introduction of additional assets has on the mean-variance frontier. If the mean-variance frontier of the benchmark assets and the frontier of the benchmark plus the new assets have exactly one point in common, this is known as *intersection.* This means that there is one mean-variance utility function for which there is no benefit from adding the new assets. If the mean-variance frontier of the benchmark assets plus the new assets coincides with the frontier of the benchmark assets only, there is *spanning*. This means that no mean-variance investor can benefit from adding the new assets to his (optimal) portfolio of the benchmark assets only. For instance, DeSantis (1995) and Cumby & Glen (1990) consider the question whether US-investors can benefit from international diversification. Taking the viewpoint of a US-investor who initially only invests in the US, these authors study the question whether they can enhance the mean-variance characteristics of their portfolio by also investing in other (developed) markets. Similarly, taking the perspective of a US-investor who invests in the US and (possibly) in other developed markets such as Japan and Europe, DeSantis (1994) and Bekaert & Urias (1996), e.g., investigate whether the investors can improve upon their mean-variance portfolio by investing in emerging markets. As a final example, Glen & Jorion (1993) investigate whether mean-variance investors with a well-diversified international portfolio of stocks and bonds should add currency futures to their portfolio, i.e., whether or not they should hedge the currency risk that arises from their positions in stocks and bonds.

As shown by DeSantis (1994), Ferson, Foerster, & Keim (1993) and Bekaert & Urias (1996), the question of mean-variance spanning and intersection can also be posed in terms of the volatility bounds introduced by Hansen & Jagannathan (1991). In that case, the interest is in the question whether a set of additional assets contains information about the volatility of the pricing kernel or the stochastic discount factor that is not already present in the initial set of assets considered by the economist. For instance, in the case of emerging markets, the question is whether considering returns from the US-market together with returns from emerging markets produces tighter volatility bounds on the stochastic discount factor than returns from the US-market only.

It turns out that there is a very close link between mean-variance frontiers and volatility bounds for the stochastic discount factors. This duality will be the subject of the next section. The analysis provided in that section will then allow us to study the question of mean-variance spanning and intersection both in terms of mean-variance frontiers and in terms of volatility bounds. The concept of mean-variance spanning and intersection will formally be introduced in Section 2.3. In that section it will be also be shown how simple regression techniques can be used to test for mean-variance spanning and intersection. In Section 2.4 we will consider how conditioning information can be incorporated in the test procedures. In Section 2.5 we will show how deviations from mean-variance intersection and spanning can be interpreted in terms of performance measures like Jensen's alpha and the Sharpe ratio, and how the regression tests for intersection can be used to derive the new optimal portfolio weights. A brief discussion of the specification error bound introduced by Hansen & Jagannathan (1997) and how this is related to mean-variance intersection will be given in Section 2.6. As with the performance measures in Section 2.5, specification error bounds are especially of interest when there is no intersection. Finally, in Section 2.7 we will illustrate the ideas introduced in Section 2.2 through 2.6 with some applications that have recently received a lot of attention in the literature. This chapter will end with a summary.

2.2 Volatility bounds and the duality with mean-variance frontiers

The purpose of this section is to give an introduction to volatility bounds and mean-variance frontiers and to show the duality between these two frontiers. Because mean-variance spanning and intersection can be defined from volatility bounds as well as from mean-variance frontiers, this section provides a basis for the analysis of mean-variance spanning and intersection in the remainder of this chapter.

2.2.1 Volatility bounds

Suppose an investor chooses his portfolio from a set of K assets, with current prices given by the K-dimensional vector P_t and whose payoffs in the next period are given by the vector P_{t+1} (including dividends and the like). Returns $R_{i,t+1}$ are payoffs with prices equal to one, i.e., $R_{i,t+1} \equiv P_{i,t+1}/P_{i,t}$. Assuming there are no market frictions such as short sales constraints and transaction costs and assuming that the law of one price holds, there exists a *stochastic discount factor* or *pricing kernel*, M_{t+1}, such that[2]

$$E[M_{t+1}R_{t+1} \mid I_t] = \iota_K, \tag{2.1}$$

where ι_K is a K-dimensional vector containing ones, and I_t is the information set that is known to the investor at time t. In the sequel we will use $E_t[.]$ as shorthand for $E[. \mid I_t]$.

One way to motivate (2.1) is to look at the discrete time consumption and portfolio problem that an investor solves:

$$\begin{aligned} & \max_{\{w_t,C_t\}} E_t[\sum_{j=0}^{\infty} \rho^j U(C_{t+j})], && (2.2)\\ \text{s.t. } W_{t+j+1} &= w'_{t+j}R_{t+j+1}(W_{t+j} - C_{t+j}), \\ w'_{t+j}\iota_K &= 1, \; j = 0, 1, 2, ... \end{aligned}$$

where C_t is consumption at time t, W_t is the wealth owned by the investor at time t, ρ is the subjective discount factor of the investor, and w_t is the K-dimensional vector of portfolio weights that the investor has to choose. The function $\mathcal{U}(C_t, C_{t+1}, ...) = \sum_{j=0}^{\infty} \rho^j U(C_{t+j})$ is a strictly increasing and concave time-separable utility function. The first order conditions of problem (2.2) imply that a valid stochastic discount factor is

$$M_{t+1} = \rho\frac{U'(C_{t+1})}{U'(C_t)} \mid_{C_t^{opt}, w_t^{opt}},$$

with $U'(.)$ being the first derivative of U. Thus, one way to think about the stochastic discount factor or pricing kernel is as the intertemporal marginal rate of substitution (IMRS).

In many of the problems we consider in this thesis, it is convenient to look at a more simple portfolio problem. Usually, we will restrict ourselves to one-period portfolio problems, where the agent maximizes his indirect utility of wealth function (see, e.g., Ingersoll (1987, p.66)):

$$\begin{aligned} & \max_{\{w\}} E_t[u(W_{t+1})], \\ \text{s.t. } W_{t+1} &= W_t w' R_{t+1}, \end{aligned}$$

[2] If, instead of the law of one price, we would impose the stronger condition that there are no arbitrage opportunities, then we would also have that $M_{t+1} > 0$.

$$w'\iota_K = 1.$$

In this case a valid stochastic discount factor is $W_t u'(W_{t+1})/\eta$, with $u'(.)$ being the first derivative of the indirect utility function evaluated in the optimal portfolio choice, and η the Lagrange multiplier for the restriction that $w'\iota_K = 1$.

The expectation of the stochastic discount factor will be denoted by v_t, i.e., $v_t \equiv E_t[M_{t+1}]$. The name *stochastic discount factor* refers to the fact that M_{t+1} discounts payoffs differently in different states of the world. To illustrate this, using the definition of covariance, (2.1) can be rewritten as

$$\iota_K = E_t[M_{t+1}R_{t+1}] = v_t E_t[R_{t+1}] + Cov_t[R_{t+1}, M_{t+1}]. \tag{2.3}$$

The first term in (2.3) uses v_t to discount the expected future payoffs, while the second term is a risk adjustment (recall that ι_K is the price-vector of the returns R_{t+1}). Accordingly, risk premia are determined by the covariance of asset payoffs with M_{t+1}. If one of the assets is a risk free asset with return R_t^f, then it follows readily from (2.1) that $R_t^f = 1/v_t$. In the sequel we will usually not impose the presence of such a risk free asset. If a risk free asset is available however, then we can always substitute $1/R_t^f$ for v_t.

Equation (2.1) is the starting point for most asset pricing models. In fact, differences in asset pricing models can be interpreted as differences in the function that each model assigns to M_{t+1} (see, e.g., Cochrane (1997)). Since each valid stochastic discount factor has to satisfy (2.1), observed asset returns can be used to derive information about these discount factors. For instance, following Hansen & Jagannathan (1991) it is possible to derive a lower bound on the variance of M_{t+1}, that each valid stochastic discount factor has to satisfy, which is known as the *volatility bound*. In this thesis, the expectation of the stochastic discount factor will usually be a free parameter. We will denote all discount factors that satisfy (2.1) and that have expectation v with $M(v)_{t+1}$, and derive a lower bound on the variance of each $M(v)_{t+1}$.

Let the expectation and covariance matrix of the returns R_{t+1} be given by μ_R and Σ_{RR} respectively, and assume that all returns are independently and identically distributed (*i.i.d.*), so that the expectations and covariances do not vary over time. This assumption will be relaxed in Section 4 of this chapter. Given the set of asset returns R_{t+1}, let $m_R(v)_{t+1}$ be a candidate stochastic discount factor that has expectation v and that is linear in the asset returns:

$$m_R(v)_{t+1} = v + \alpha(v)'(R_{t+1} - \mu_R), \tag{2.4}$$

where we write $\alpha(v)$ to indicate that these coefficients are a function of the expectation of $M(v)_{t+1}$. Substituting (2.4) into (2.1) gives for $\alpha(v)$:

$$\alpha(v) = \Sigma_{RR}^{-1}(\iota_K - v\mu_R). \tag{2.5}$$

Since both $M(v)_{t+1}$ and $m_R(v)_{t+1}$ satisfy (2.1) we have that $E[(M(v)_{t+1} - m_R(v)_{t+1})R_{t+1}] = 0$, so the difference between any $M(v)_{t+1}$ that satisfies (2.1) and $m_R(v)_{t+1}$ is orthogonal to R_{t+1} and therefore to $m_R(v)_{t+1}$ itself. This implies for the variance of $M(v)_{t+1}$ that:

$$\begin{aligned} Var[M(v)_{t+1}] &= Var[m_R(v)_{t+1}] + Var[(M(v)_{t+1} - m_R(v)_{t+1})] \\ &\geq Var[m_R(v)_{t+1}], \end{aligned} \tag{2.6}$$

which shows that $m_R(v)_{t+1}$ has the lowest variance of all valid stochastic discount factors $M(v)_{t+1}$. This minimum variance can be obtained by combining (2.4) and (2.5):

$$Var[m_R(v)_{t+1}] = (\iota_K - v\mu_R)'\Sigma_{RR}^{-1}(\iota_K - v\mu_R). \tag{2.7}$$

Thus, any pricing model that aims to price the assets R_{t+1} correctly, has to yield a pricing kernel that, for a given v, has a variance at least as large as (2.7). Equivalently, if we know that agents choose their optimal portfolio from the assets that are in R_{t+1}, then (2.7) gives the minimum amount of variation of their IMRS that is needed to be consistent with the distribution of asset returns. Luttmer (1996) extends this kind of analysis taking into account market frictions such as short sales constraints and transaction costs. For that case, results similar to his will be derived in Chapter 4. For the frictionless markets setting, Snow (1991) provides a similar analysis to derive bounds on other moments of the discount factor as well.

2.2.2 Duality between volatility bounds and mean-variance frontiers

So far we have focussed on some of the implications of Equation (2.1) and the distribution of asset returns for any asset pricing model or utility function, i.e., for any choice of the stochastic discount factor $M(v)_{t+1}$. Specifically, we derived the minimum amount of variation in stochastic discount factors that is needed to be consistent with the distribution of asset returns. In this section we will show that there is a close correspondence between these volatility bounds and mean-variance frontiers and that stochastic discount factors that correspond to mean-variance optimizing behavior are the stochastic discount factors with the lowest volatility. Mean-variance optimizing behavior is a special case of the portfolio problem considered before, where the problem the agent faces is $\max_{\{w\}} E[u(W_{t+1})]$, and where $E[u(.)]$ is of the form $f(w'\mu_R, w'\Sigma_{RR}w)$, with f increasing in its first argument and decreasing in its second argument.

For further reference it is useful to define the efficient set variables:

$$A \equiv \iota_K' \Sigma_{RR}^{-1} \iota_K, \; B \equiv \mu_R' \Sigma_{RR}^{-1} \iota_K, \text{ and } C \equiv \mu_R' \Sigma_{RR}^{-1} \mu_R.$$

A mean-variance efficient portfolio w^* is the solution to the problem

$$\max_{\{w\}} L = w'\mu_R - \gamma w' \Sigma_{RR} w - \eta(w'\iota_K - 1),$$

where γ is the coefficient of risk aversion. From the first order conditions of this problem it follows that a portfolio w^* is mean-variance efficient if there exist scalars γ and η such that[3]

$$w^* = \gamma^{-1} \Sigma_{RR}^{-1} (\mu_R - \eta \iota_K). \tag{2.8}$$

Because of the restriction $w'\iota_K = 1$, it also follows that $\gamma = B - A\eta$, implying that each mean-variance efficient portfolio is uniquely determined when either γ or η is known, unless $\eta = B/A$. It is straightforward to show that for a given mean-variance efficient portfolio w^*, the Lagrange multiplier η equals the expected return on the zero-beta portfolio of w^*, i.e., the intercept of the line tangent to the mean-variance frontier at w^* (in mean-standard deviation space). Since B/A, is the expected return on the global minimum variance (GMV) portfolio, this is the intercept of the asymptotes of the mean-variance frontier, but there are no lines tangent to the frontier originating at this point (see, e.g., Ingersoll (1987, p.86)).

To show the duality between mean-variance frontiers and volatility bounds, take $\alpha(v)$ for a given v, and choose a mean-variance efficient portfolio such that $\eta = 1/v$. It follows from (2.8) and (2.5) that

$$w^*(v) = \frac{\Sigma_{RR}^{-1}(\mu_R - \frac{1}{v}\iota_K)}{B - \frac{1}{v}A} = \frac{\Sigma_{RR}^{-1}(\iota_K - v\mu_R)}{A - vB} = \frac{\alpha(v)}{\iota_K'\alpha(v)}, \tag{2.9}$$

which shows that the vector $\alpha(v)$ is proportional to a mean-variance efficient portfolio with zero-beta return equal to $1/v$. Thus, each point on the volatility bound of stochastic discount factors, i.e., each $\alpha(v)$, corresponds to a unique point on the mean-variance frontier, i.e., a unique $w^*(v)$. The only exception to this result is the case where $\iota_K'\alpha(v) = 0$, which is the case if $v = A/B$, or equivalently, $\eta = B/A$. As already noted, this is the case where the zero-beta return equals the expected return on the global minimum variance portfolio (see also Hansen & Jagannathan (1991)). The duality between the mean-variance frontier of R_{t+1} and the volatility bound derived from R_{t+1} can also be seen directly from (2.5) and (2.8). Comparing the coefficients $\alpha(v)$ for the minimum variance stochastic discount factor in (2.5) and the portfolio weights w^* in (2.8) for

[3] More precisely, these are the minimum variance portolios, i.e., the portfolios that have minimum variance for a given expected return. The mean-variance efficient portfolios, i.e., the portfolios that also have maximum expected return for a given variance, require in addition that $\gamma \geq 0$.

$\eta = 1/v$, it can be seen immediately that the coefficients $\alpha(v)$ are proportional to the portfolio weights w^*, where the coefficient of proportionality is equal to $-\eta/\gamma$, i.e., $w^* = (-\eta/\gamma)\alpha(v)$.

Summarizing, finding stochastic discount factors that have the lowest variance of all stochastic discount factors that price a set of asset returns R_{t+1} correctly is tantamount to finding mean-variance efficient portfolios for these same assets R_{t+1}. It should be clear by now however, that the interpretation of these two problems is entirely different. The volatility bound is concerned with the properties of marginal rates of substitution (or asset pricing models) that can be derived from the distribution of asset returns. Specifically, given a set of asset returns which we know that are available to investors, the problem is to determine the minimum amount of variation that each investors IMRS must have to be consistent with the distribution of asset returns. The mean-variance frontier on the other hand, is concerned with the properties of optimal portfolios: given that an investor is only concerned about expected portfolio return and the variance of this return, the problem is to determine the mean-variance properties of optimal portfolios. In the remainder of this chapter we will study the effects of adding new assets to the set of assets available to investors. Although most of the results will be stated in terms of mean-variance frontiers and mean-variance efficient portfolios, it should be kept in mind that there is always a dual interpretation in terms of volatility bounds.

2.3 Mean-variance spanning and intersection

In the previous section we considered the volatility bounds and mean-variance frontiers that can be derived from a given set of K assets with return vector R_{t+1}. Suppose now that an investor takes an additional set of N assets with return vector r_{t+1} into account in his portfolio problem. The question we are interested in is under what conditions mean-variance efficient portfolios derived from the set of returns R_{t+1} are also mean-variance efficient for the larger set of $K + N$ assets (R_{t+1}, r_{t+1}). This problem was addressed in the seminal paper of Huberman & Kandel (1987). If there is only one value of γ or η for which mean-variance investors can not improve their mean-variance efficient portfolio by including r_{t+1} in his investment set, the mean-variance frontiers of R_{t+1} and (R_{t+1}, r_{t+1}) have exactly one point in common, which is referred to as *intersection*. In this case we will say that the mean-variance frontier of R_{t+1} intersects the mean-variance frontier of (R_{t+1}, r_{t+1}), or simply that R_{t+1} intersects (R_{t+1}, r_{t+1}). If there is no mean-variance investor that can improve his mean-variance efficient portfolio by including r_{t+1} in his investment set, the mean-variance frontiers of R_{t+1} and (R_{t+1}, r_{t+1}) coincide, which is referred to as *spanning*. In this case we will say that (the mean-variance frontiers of) R_{t+1} spans (the mean-variance frontier of) (R_{t+1}, r_{t+1}). As suggested by the previous section, and as shown by Ferson, Foerster, & Keim

(1993), DeSantis (1994), and Bekaert & Urias (1996), the concept of mean-variance spanning and intersection has a dual interpretation in terms of volatility bounds. In terms of volatility bounds mean-variance spanning means that the volatility bound derived from the returns R_{t+1} is the same as the bound derived from (R_{t+1}, r_{t+1}). Therefore, the minimum variance stochastic discount factors for R_{t+1}, $m_R(v)_{t+1}$, are also the minimum variance stochastic discount factors for (R_{t+1}, r_{t+1}), and the asset returns r_{t+1} do not provide information about the necessary volatility of stochastic discount factors that is not already present in R_{t+1}. As will be shown formally below, mean-variance intersection is equivalent to saying that the volatility bounds derived from R_{t+1} and (R_{t+1}, r_{t+1}) have exactly one point in common. Thus, in case of intersection there is exactly one value of v for which the minimum variance stochastic discount factor does not change, whereas for all other values of v it does.

In finite samples it will in general be the case that adding assets causes a shift in the estimated mean-variance frontier and the estimated volatility bound. This shift may very well be the result of estimation error however, and the main question is whether the observed shift is too large to be attributed to chance. Therefore, to answer the question whether or not the observed shift in the mean-variance frontier is significant in statistical terms, in this section we will also show how regression analysis can be used to test for spanning and intersection.

2.3.1 Spanning and intersection in terms of mean-variance frontiers

To state the problem formally, the hypothesis of mean-variance intersection means that there is a portfolio w^* which is mean-variance efficient for the smaller set R_{t+1} and which is also mean-variance efficient for the larger set (R_{t+1}, r_{t+1}). In the sequel, variables that refer to the smaller set R_{t+1} (r_{t+1}) will be referred to with a subscript R (r), or with their dimension K (N), whereas variables that refer to the larger set (R_{t+1}, r_{t+1}), will not have any subscript or will have their dimension as subscript, $K+N$. Thus, w_R is a K-dimensional vector with portfolio weights for the assets in R_{t+1}, and w is a $(K+N)$-dimensional vector with portfolio weights for all the available assets (R_{t+1}, r_{t+1}). The hypothesis of mean-variance intersection comes down to the statement that there exists a mean-variance efficient portfolio w^* of the form

$$w^* = \begin{pmatrix} w_R^* \\ 0_N \end{pmatrix}, \tag{2.10}$$

i.e., there exist scalars γ and η, such that

$$\mu - \eta \iota_{K+N} = \gamma \Sigma \begin{pmatrix} w_R^* \\ 0_N \end{pmatrix}. \tag{2.11}$$

If such a portfolio w^* exists, there is one point on the mean-variance frontier of R_{t+1} that also lies on the mean-variance frontier of (R_{t+1}, r_{t+1}). Using obvious notation, μ consists of two subvectors μ_R and μ_r, and Σ consists of submatrices Σ_{RR}, Σ_{Rr}, Σ_{rR}, and Σ_{rr}. The first K rows of (2.11) imply that

$$\mu_R - \eta\iota_K = \gamma\Sigma_{RR}w_R^* \Leftrightarrow w_R^* = \gamma^{-1}\Sigma_{RR}^{-1}(\mu_R - \eta\iota_K). \tag{2.12}$$

For one thing, note that (2.12) simply says that w_R^* is indeed mean-variance efficient for the smaller set R_{t+1}.

We will now derive the restrictions on the distribution of R_{t+1} and r_{t+1} that are equivalent to mean-variance intersection. In order to do so, substitute (2.12) in the last N rows of (2.11) to obtain:

$$\begin{aligned} \mu_r - \eta\iota_N &= \Sigma_{rR}\Sigma_{RR}^{-1}(\mu_R - \eta\iota_K), \Leftrightarrow \\ &(\mu_r - \beta\mu_R) + (\beta\iota_K - \iota_N)\eta = 0, \end{aligned} \tag{2.13}$$

with $\beta \equiv \Sigma_{rR}\Sigma_{RR}^{-1}$. Thus, if there is a portfolio that is mean-variance efficient for the smaller set R_{t+1} that is also mean-variance efficient for the larger set (R_{t+1}, r_{t+1}), there must exist a η such that the restriction in (2.13) holds. It follows immediately from the derivation above that this η is the zero-beta return that corresponds to the portfolio w_R^* (and w^*).

If there is mean-variance spanning *all* mean-variance efficient portfolios w^* must be of the form (2.10), i.e., (2.11) must be true for *all* values of η and the corresponding γ's. Going through the same steps, if (2.11) must hold for all η, (2.13) must hold for all η, and this can only be the case if

$$\mu_r - \beta\mu_R = 0 \text{ and } \beta\iota_K - \iota_N = 0, \tag{2.14}$$

which are the restrictions imposed by the hypothesis of spanning. If these restrictions on the distribution of R_{t+1} and r_{t+1} hold, every point on the mean-variance frontier of R_{t+1} is also on the mean-variance frontier of (R_{t+1}, r_{t+1}) and the two frontiers coincide.

2.3.2 Spanning and intersection in terms of volatility bounds

In the previous section we defined mean-variance spanning and intersection from the properties of mean-variance efficient portfolios and we derived the equivalent restrictions on the distribution of asset returns. In this section we analyze mean-variance intersection and spanning from the properties of minimum variance stochastic discount factors that price the assets in R_{t+1} and in (R_{t+1}, r_{t+1}) correctly and we show that this imposes the same restrictions on the distribution of the asset returns. In terms of volatility bounds, the hypothesis of intersection is that there is a value of v such that the minimum variance stochastic discount factor for R_{t+1}, i.e., $m_R(v)_{t+1}$, is also the minimum variance stochastic discount factor for the larger set (R_{t+1}, r_{t+1}). The discount factor $m_R(v)_{t+1}$ as defined by (2.4) and (2.5) is the minimum variance stochastic discount factor for this larger set if it also prices r_{t+1} correctly. If $m_R(v)_{t+1}$ prices both R_{t+1} and r_{t+1} correctly, the difference between $m_R(v)_{t+1}$ and any other $M(v)_{t+1}$ that prices R_{t+1} and r_{t+1} correctly is orthogonal to R_{t+1} and r_{t+1}, implying that $m_R(v)_{t+1}$ must have the lowest variance among all stochastic discount factors $M(v)_{t+1}$, by the same reasoning that leads to (2.6).

Thus, the hypothesis of intersection for volatility bounds can be stated as:

$$\exists v \text{ s.t. } E[r_{t+1} m_R(v)_{t+1}] = \iota_N. \tag{2.15}$$

To show that this hypothesis imposes the same restrictions on the distribution of R_{t+1} and r_{t+1} as in (2.13), substitute (2.4) and (2.5) into (2.15):

$$\begin{aligned} E[r_{t+1}(v + (R_{t+1} - \mu_R)'\Sigma_{RR}^{-1}(\iota_K - v\mu_R))] &= \iota_N, \Leftrightarrow \\ (\mu_r - \Sigma_{rR}\Sigma_{RR}^{-1}\mu_R)v + (\Sigma_{rR}\Sigma_{RR}^{-1}\iota_K - \iota_N) &= 0, \Leftrightarrow \\ (\mu_r - \beta\mu_R)v + (\beta\iota_K - \iota_N) &= 0. \end{aligned} \tag{2.16}$$

Dividing both sides of (2.16) by v shows that the hypothesis of intersection in terms of volatility bounds indeed implies the same restrictions as the hypothesis of intersection in terms of mean-variance frontiers, if we choose $\eta = 1/v$. This could be expected beforehand, since from the duality between mean-variance frontiers and volatility bounds in (2.9) we already knew that the vector $\alpha_R(v)$ that defines $m_R(v)_{t+1}$, is proportional to a mean-variance efficient portfolio with zero-beta return $\eta = 1/v$. The hypothesis that w^* is of the form $(w_R^{*\prime}\ 0_N')'$ is therefore equivalent the hypothesis that $\alpha(v)$ is of the form $(\alpha_R(v)'\ 0_N')'$.

By the same logic, the hypothesis of spanning in terms of volatility bounds, requires that $m_R(v)_{t+1}$ prices the returns r_{t+1} for *all* values of v:

$$E[r_{t+1} m_R(v)_{t+1}] = \iota_N, \quad \forall v, \tag{2.17}$$

since in that case the entire volatility bound derived from (R_{t+1}, r_{t+1}) coincides with the volatility bound derived from (R_{t+1}) only. This requirement implies that (2.16) holds for all values of v, and this can only be the case if the restrictions in (2.14) hold.

2.3.3 Intersection and mean-variance efficiency of a given portfolio

A question that is of obvious interest both from a portfolio choice perspective and from an asset pricing perspective, is the question whether or not a given portfolio w^p is mean-variance efficient or not. From a portfolio choice perspective, an investor will be interested in whether or not his portfolio has the desired properties of a mean-variance efficient portfolio. From an asset pricing perspective, the most interesting question is often whether or not the market portfolio is mean-variance efficient as the CAPM predicts.

Denote the return on some portfolio w^p by R^p_{t+1} and its expectation by μ^p. The question whether or not w^p is mean-variance efficient with respect to the $N+1$ assets (R^p_{t+1}, r_{t+1}), is obviously a special case of the question whether or not there is mean-variance intersection with $K = 1$ and $R_{t+1} = R^p_{t+1}$, since intersection in this case simply means that the portfolio w^p is on the mean-variance frontier of (R^p_{t+1}, r_{t+1}). Therefore, if w^p is mean-variance efficient for the set (R^p_{t+1}, r_{t+1}), the following restrictions on the distribution of R^p_{t+1} and r_{t+1} should hold:

$$\mu_r = \eta\iota_N + \beta^p(\mu^p - \eta), \tag{2.18}$$

where β^p is the N-dimensional vector $Cov[r_{t+1}, R^p_{t+1}]/Var[R^p_{t+1}]$, and $\mu^p = E[R^p_{t+1}]$. When testing for mean-variance efficiency, R^p_{t+1} is usually the return on a portfolio of r_{t+1}.

What we want to establish in this section however, is that the hypothesis that the mean-variance frontier of R_{t+1} ($K \geq 1$) intersects the frontier of (R_{t+1}, r_{t+1}) at a given value of $\eta = 1/v$, is tantamount to the hypothesis that the portfolio w^*_R that is mean-variance efficient for R_{t+1} and that has η as its zero-beta rate is also mean-variance efficient with respect to (R_{t+1}, r_{t+1}). Denote the return on w^*_R as R^*_{t+1} and its expectation as μ^*. Recall that the portfolio w^*_R is given by the first K rows of (2.11)

$$w^*_R = \gamma^{-1}\Sigma^{-1}_{RR}(\mu_R - \eta\iota_K),$$

from which

$$w^{*\prime}_R(\mu_R - \eta\iota_K) = \gamma w^{*\prime}_R\Sigma_{RR}w^*_R \Leftrightarrow \gamma = \frac{\mu^* - \eta}{Var[R^*_{t+1}]}.$$

Substituting these relations into (2.13) an defining $\beta^* \equiv Cov[r_{t+1}, R^*_{t+1}]/Var[R^*_{t+1}]$, results in

$$\begin{aligned} 0 \;=\;& (\mu_r - \eta\iota_N) - \Sigma_{rR}\Sigma^{-1}_{RR}(\mu_R - \eta\iota_K) = (\mu_r - \eta\iota_N) - \gamma\Sigma_{rR}w^*_R = \\ & (\mu_r - \beta^*\mu) + (\beta^* - \iota_N)\eta. \end{aligned} \tag{2.19}$$

These are the same restrictions as (2.18). Thus, the hypothesis of intersection implies the same restrictions on the distribution of R_{t+1} and r_{t+1} as the hypothesis that w^*_R is mean-variance efficient with respect to r_{t+1}, as could be expected beforehand.

Although the hypothesis of intersection and the hypothesis of mean-variance efficiency impose the same restrictions, the problems underlying these hypotheses are still different. From a portfolio perspective, the intersection hypothesis often stems from the question whether there is *a* mean-variance efficient portfolio of R_{t+1} that is also mean-variance efficient for (R_{t+1}, r_{t+1}), without specifying this portfolio beforehand. In other words, the problem is whether it is sufficient to invest in R_{t+1} only, or whether the investor should invest in r_{t+1} as well. In testing the hypothesis that a given portfolio is mean-variance efficient on the other hand, the investor takes a *known* portfolio and he wants to know whether this is optimal or whether it should be adjusted by including assets from r_{t+1} as well.

From an asset pricing perspective the difference between tests for intersection and tests for mean-variance efficiency is less clear cut and is motivated by different asset pricing models. For instance, the CAPM implies that the market portfolio is mean-variance efficient. Exact arbitrage pricing on the other hand, implies that there exist a number of factor mimicking portfolios corresponding to the factors in the arbitrage pricing model, and that the mean-variance frontier of these factor mimicking portfolios intersects the mean-variance frontier of these same portfolios plus all other assets in the economy (see, e.g., Huberman, Kandel, & Stambaugh (1987)).

2.3.4 Testing for spanning and intersection

So far we derived the restrictions implied by the hypotheses of mean-variance intersection and spanning for the distribution of R_{t+1} and r_{t+1}. Huberman & Kandel (1987) showed how regression can be used to test these hypotheses. To see how regression can be used to test for intersection, start from (2.13):

$$\mu_r - \eta\iota_N = \beta(\mu_R - \eta\iota_K).$$

Replacing expected returns μ_r and μ_R with realized returns r_{t+1} and R_{t+1}, gives the regression

$$r_{t+1} = \alpha + \beta R_{t+1} + u_{t+1}, \tag{2.20}$$

with $\alpha = \mu_r - \beta\mu_R$, $\varepsilon_{t+1} = u_{r,t+1} - \beta u_{R,t+1}$, $u_{r,t+1} \equiv r_{t+1} - \mu_r$ and $u_{R,t+1} \equiv R_{t+1} - \mu_R$. It can readily be checked that under the null hypotheses of spanning and intersection $Cov[\varepsilon_{t+1}, R_{t+1}] = 0$. Notice that α is a N-dimensional vector of intercepts, β is a $N \times K$-dimensional matrix of slope coefficients, and ε_{t+1} is a N-dimensional vector of error terms. The restrictions imposed by the hypothesis of intersection in (2.13) can now be stated as

$$\alpha - \eta(\iota_N - \beta\iota_K) = 0. \tag{2.21}$$

It should be clear from the previous section that with intersection there are two cases of interest. First, we may be interested in testing for intersection for a given value of the zero-beta rate η. In that case the restrictions in (2.21) should hold for this specific value of η, which is a set of linear restrictions. In the sequel we will mainly be interested in this case. Second, the interest may be in the question whether there is intersection at some unknown point of the frontier, i.e., for some unknown value of η. In that case the hypothesis is that there exists some η such that the restrictions in (2.21) hold. This hypothesis can be stated as

$$\alpha_i/(1-\beta_i) = \alpha_j/(1-\beta_j),\ i,j = 1,...,N.$$

Thus, the hypothesis that there is intersection at some point of the frontier imposes a set of nonlinear restrictions on the regression parameters in (2.20). Notice that given estimates of α_i and β_i an estimate of the zero-beta rate for which there is intersection can be obtained from $\alpha_i/(1-\beta_i)$. Also note, that testing whether there is intersection at some unknown point of the frontier only makes sense if $N \geq 2$, since there is always intersection if $N = 1$.

Recall that the hypothesis of spanning implies that (2.13) holds for all values of η. Therefore, going through the same steps, the restrictions imposed by the hypothesis of spanning can be stated as

$$\alpha = 0 \text{ and } \beta\iota_K - \iota_N = 0. \tag{2.22}$$

The restrictions in terms of the regression model in (2.20) are intuitively very clear. For instance, the spanning restrictions in (2.22) state that if there is spanning, then each return of the additional assets, $r_{i,t+1}$, $i = 1,2,...,N$, can be written as the return of a portfolio of the benchmark assets $\beta_i R_{t+1}$, $\beta_i\iota_K = 1$, plus an error term $\varepsilon_{i,t+1}$ which has expectation zero and which is orthogonal to the returns R_{t+1}. Since such an asset can only add to the variance of portfolios of R_{t+1}, and not to the expected return, mean-variance optimizing agents will not include such an asset in their portfolio. A similar interpretation holds for the intersection restrictions.

If the returns series R_{t+1} and r_{t+1} are stationary and ergodic, consistent estimates of the parameters α and β in (2.20) are easily obtained using OLS. In writing down the test statistics for (2.21) and (2.22), it is convenient to use a different specification of (2.20), in which all the coefficients α and β are stacked into one big vector:

$$r_{t+1} = (I_N \otimes \begin{pmatrix} 1 & R'_{t+1} \end{pmatrix})\, b + \varepsilon_{t+1}, \tag{2.23}$$

where $b = vec\left(\begin{pmatrix} \alpha & \beta \end{pmatrix}'\right)$, a $(K+1)N$-dimensional vector. If $\widehat{b}$ is the OLS estimate of b and $\widehat{Q}$ is a consistent estimate of the asymptotic covariance matrix of $\widehat{b}$, the hypotheses of intersection and spanning can be tested using a standard Wald test. Defining

$$H(\eta)_{int} \equiv I_N \otimes \begin{pmatrix} 1 & \eta\iota'_K \end{pmatrix} \text{ and} \tag{2.24a}$$

$$h(\eta)_{int} \equiv H(\eta)_{int}\widehat{b} - \eta\iota_N, \tag{2.24b}$$

the Wald test-statistic for intersection can be written as

$$\xi_W^{int} = h(\eta)'_{int}\left(H(\eta)_{int}\widehat{Q}H(\eta)'_{int}\right)^{-1}h(\eta)_{int}. \tag{2.25}$$

Similarly, defining

$$H_{span} \equiv I_N \otimes \begin{pmatrix} 1 & 0'_K \\ 0 & \iota'_K \end{pmatrix} \text{ and} \tag{2.26a}$$

$$h_{span} \equiv H_{span}\widehat{b} - I_N \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.26b}$$

the Wald test-statistic for spanning can be written as

$$\xi_W^{span} = h'_{span}\left(H_{span}\widehat{Q}H'_{span}\right)^{-1}h_{span}. \tag{2.27}$$

Under the null hypotheses and regularity conditions, the limit distribution of ξ_W^{int} will be χ^2_N and the limit distribution of ξ_W^{span} will be χ^2_{2N}. The test statistics in (2.25) and (2.27) have interesting economic interpretations in terms of performance measures. The relationship between tests for intersection and spanning and performance evaluation will be discussed in detail in Section 2.5.3.

Chen & Knez (1996) and Hall & Knez (1995) propose a test for intersection that is based on (2.15). Define the deviation from the equality in (2.15) to be $\lambda(v)$:

$$\lambda(v) \equiv E[m_R(v)_{t+1}r_{t+1}] - \iota_N. \tag{2.28}$$

In Section 2.5.1 we will interpret $\lambda(v)$ scaled by v as a generalization of the well-known Jensen measure. Given an estimate of the parameters $\alpha_R(v)$ using the sample equivalent of (2.5):

$$\widehat{\alpha}_R(v) = \left(\frac{1}{T}\sum_{t=1}^{T}(R_t - \overline{R})(R_t - \overline{R})'\right)^{-1}\left(\iota_K - v\overline{R}\right),$$

with $\overline{R}$ the sample mean of R_t, define $\widehat{\lambda}(v)_t$ as

$$\widehat{\lambda}(v)_t \equiv r_t(v + \widehat{\alpha}_R(v)'(R_t - \overline{R})) - \iota_N.$$

A test for the hypothesis of intersection, $\lambda(v) = 0$, can now be based on

$$\xi_{CK}^{int} = \left(\frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda}(v)_t\right)'\left(\widehat{Var}[\widehat{\lambda}(v)_t]\right)^{-1}\left(\frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda}(v)_t\right), \tag{2.29}$$

where the estimate $\widehat{Var}[\widehat{\lambda}(v)_t]$ can for instance be obtained using the method suggested by Newey & West (1987b). The limit distribution of the test-statistic ξ_{CK}^{int} is also χ^2_N. It is straightforward to show that for $\eta = 1/v$, $\left(\frac{1}{T}\sum_{t=1}^{T}\widehat{\lambda}(v)_t\right)/v = H(\eta)_{int}\widehat{b} - h(\eta)_{int}$, and that the only difference in the

Wald test-statistic in (2.25) and the statistic proposed in (2.29) is the way in which the covariance matrix is estimated.

A disadvantage of the test originally proposed by Chen & Knez (1995) is that they test for intersection for a very specific stochastic discount factor, which corresponds to the minimum second moment portfolio. This discount factor can be found by projecting the kernel M_{t+1} on the asset returns only, excluding the constant. The corresponding portfolio on the mean-variance frontier is the one with the minimum second moment among all portfolios on the frontier, and can graphically be found as the tangency point between the mean-variance frontier and a circle with its centre at the origin. The problem with this portfolio is that it is located at the inefficient part of the frontier, implying that the test used by Chen & Knez (1995) is for intersection at an inefficient portfolio. Therefore it is economically not very interesting, unless there exists a risk free asset. Since in the test statistic in (2.29) the discount factor $m_R(v)_{t+1}$ results from a projection of M_{t+1} on R_{t+1} plus a constant, this test allows us to test for intersection at any mean-variance efficient portfolio, so this test does not suffer from the problem of the test originally suggested by Chen & Knez.

Alternative tests for the hypotheses of intersection and spanning are suggested by Huberman & Kandel (1987), who propose a likelihood ratio test, and by Snow (1991) and DeSantis (1995), who propose a Generalized Method of Moments (GMM) procedure. This latter procedure is also identical to the *region subset test* suggested by Hansen, Heaton & Luttmer (1995) which is equivalent to a test for intersection. A comparison of the small sample properties of various test-procedures can be found in Bekaert & Urias (1996). The GMM-based test or region subset test is based on the observation that under the null hypotheses of spanning or intersection, the kernel that prices R_{t+1} and r_{t+1} correctly is of the form

$$\begin{aligned} m(v)_{t+1} &= v + \alpha_R(v)'(R_{t+1} - \mu_R) + \alpha_r(v)'(r_{t+1} - \mu_r), \\ \text{with } \alpha_r(v) &= 0. \end{aligned}$$

Given that $\alpha_r(v) = 0$, a GMM-estimate of the K parameters in $\alpha_R(v)$ can be obtained by using the $K + N$ sample moments

$$g_T(a_R(v)) = \frac{1}{T}\sum_{t=1}^{T}\left\{\begin{pmatrix} R_t \\ r_t \end{pmatrix}(v + a_R(v)'(R_t - \overline{R})\right\} - \iota_{K+N} = \frac{1}{T}\sum_{t=1}^{T} g_t(a_R(v)).$$

A consistent estimate of $\alpha_R(v)$ can therefore be obtained by solving

$$\min_{a_R(v)} g_T(a_R(v))' W_T g_T(a_R(v)) = J_T(a_R(v)), \tag{2.30}$$

where W_T is a symmetric nonsingular weighting matrix. Notice that the GMM-estimate of the K parameters $\alpha_R(v)$ obtained from (2.30) is based on $K + N$ moment restrictions. The N overidentifying restrictions are derived from the hypothesis that $m_R(v)_{t+1}$ must also price the N additional assets r_{t+1}. Intersection for a given value of v can now be tested by using the fact that under the null-hypothesis and regularity conditions $TJ_T(a_R(v))$ is asymptotically χ^2_N-distributed. Since spanning implies that (2.15) holds for (at least) two different values of v, the GMM-based test can easily be extended by estimating two vectors $\alpha_R(v_1)$ and $\alpha_R(v_2)$ simultaneously ($v_1 \neq v_2$) using (2.30). In this case there are $2K$ parameters to be estimated with $2(K + N)$ moment conditions. The test for spanning is therefore a test for the $2N$ overidentifying restrictions and will asymptotically be χ^2_{2N}-distributed under the null-hypothesis of spanning.

2.3.5 An empirical illustration

To illustrate the idea of mean-variance spanning and intersection, this section provides an empirical example. We take the case of an investor that initially only invests in the Morgan Stanley Capital International (MSCI) Indices of the US and Canada. The investor subsequently also wants to invest in the MSCI Indices of the Netherlands and Japan. The available dataset consists of monthly dollar-based returns, for the period January 1985 until June 1996, giving a total of 138 observations. The data are obtained from Datastream. Summary statistics for these four indices are presented in Table 2.1.

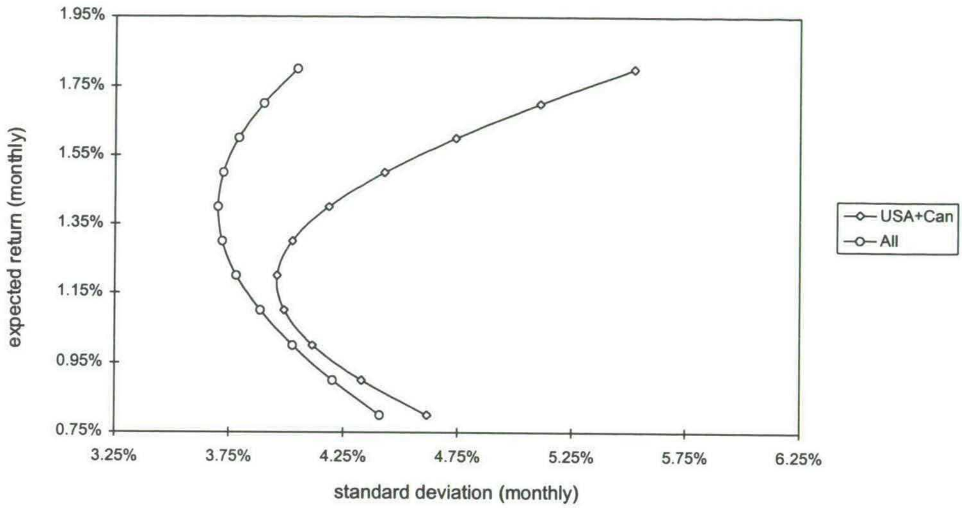
Table 2.1: Summary statistics for the four MSCI-indices

Summary statistics are provided for monthly returns on the MSCI Indices for the US, Canada, the Netherlands and Japan. All results are based on unhedged US dollar-based returns over the period from January 1985 until June 1996.

	US	Can	NL	Jap
Average	1.38%	0.86%	1.82%	1.43%
Std.dev.	4.16%	4.44%	4.41%	7.55%
	Correlations			
US	1.00	0.72	0.61	0.21
Can		1.00	0.59	0.26
NL			1.00	0.42
Jap				1.00

The mean-variance frontiers for the smaller set of the US and Canadian indices and for the larger set of all four indices are shown in Figure 2.1. Note from the graphical analysis that it appears to be the case that the two mean-variance frontiers are close to each other at the inefficient part of the frontiers, i.e., that there may be intersection but not spanning.

Figure 2.1: The figure shows mean-variance frontiers for the MSCI Indices of the US and Canada and for the MSCI Indices of the US, Canada, the Netherlands and Japan. The frontiers are based on monthly unhedged US dollar-based returns over the period from January 1985 until June 1996.



Tests whether there is intersection or spanning appear in Table 2.2. This table presents OLS-estimates of a regression of the returns of the Dutch and Japanese indices on the indices of the US and Canada, as well as Wald test-statistics whether there is intersection or spanning for the Netherlands, Japan, or both. The intersection tests are for intersection at some point and for intersection at a given annualized value of η of 3%, which corresponds to $\eta = 1.0025$ (monthly). The results in Table 2.2 show that for the Netherlands the hypotheses of intersection at $\eta = 1.0025$ and of spanning can be rejected at any conventional level of significance. For Japan the hypothesis of intersection at $\eta = 1.0025$ can not be rejected. Joint tests for intersection at $\eta = 1.0025$ for the Netherlands and Japan are rejected convincingly. This is also true for the hypothesis of spanning. However, the hypothesis of intersection at some point can not be rejected. Notice that we do not report a test for this latter hypothesis for the Netherlands or Japan only, since if only one asset is added there is always intersection at some point. The reason for this is that the mean-variance frontier of three assets always contains one point in which the portfolio weight of one of the three assets is zero, which is the intersection point.

The fact that the spanning hypothesis is rejected for both the Netherlands and Japan, means that some US-investors with a mean-variance utility function can improve their efficient set by

Table 2.2: Intersection and spanning tests

Tests are reported for the hypothesis that there is intersection or spanning of the MSCI Indices for the Netherlands and Japan by the MSCI Indices of the US and Canada. The first part of the table shows OLS-regression estimates with associated standard errors in parentheses. In the latter part the values in parentheses are probability levels associated with the reported test-statistics. All results are based on unhedged US dollar-based returns over the period from January 1985 until June 1996.

	Regression estimates			
	NL		Jap	
Const.	0.293	(0.073)	0.539	(0.158)
US	0.407	(0.100)	0.086	(0.216)
Can	0.309	(0.093)	0.385	(0.202)
	Intersection at some point			
	NL	Jap	Both	
Wald			1.15	
(p-value)			(0.283)	
	Intersection for $\eta = 1.0025$			
	NL	Jap	Both	
Wald	9.64	1.73	9.55	
(p-value)	(0.002)	(0.189)	(0.008)	
	Spanning tests			
	NL	Jap	Both	
Wald	20.61	11.78	24.91	
(p-value)	(0.000)	(0.003)	(0.000)	

investing in the Netherlands and Japan besides the US and Canada. The fact that we can not reject the hypothesis of intersection at some point means that there is (at least) one value of the risk aversion coefficient for US-investors, for which we can not reject the hypothesis that they can not improve the efficiency of their portfolio by investing in the Netherlands and Japan besides their investments in the US and Canada. An indication of the economic significance of these results can be obtained from the mean-variance frontiers. For instance, at a standard deviation of 4.0% per month, including the Netherlands and Japan in the portfolio, gives a nontrivial increase in expected return from 1.25% to 1.75% per month. In Section 2.5.4 we will give an interpretation of these test results in terms of performance measures.

2.4 Testing for spanning and intersection with conditioning information

The purpose of this section is to incorporate conditioning information in tests for intersection and spanning. Until now we assumed that returns are independently and identically distributed (*i.i.d.*). However, there is ample evidence that asset returns are to some extent predictable. For instance, stock and bond returns can be predicted from variables like lagged returns, dividend yields, short term interest rates, and default premiums (see, e.g., Ferson (1995)) and futures returns can be predicted from hedging pressure variables (see Chapter 6 e.g.) as well as from the spread between spot and forward prices. In Section 2.4.1 we will show how conditional information can be used in a straightforward way by using *scaled returns* (see, e.g., Cochrane (1997) and Bekaert & Urias (1996)). Although this is a fairly general and intuitive way of incorporating conditional information, a disadvantage of this method is that the number of returns - and therefore the dimension of the estimation and testing problem - increases quickly. In Section 2.4.2 we show that this problem can be circumvented if it is assumed that variances and covariances are constant, while expected returns are allowed to vary over time. Under this assumption it is shown that the conditioning variables can easily be accounted for by using them as additional regressors. The restrictions for the intersection and spanning hypotheses appear to be very similar to the restrictions in case returns are independently and identically distributed. This way of incorporating conditional variables also has the additional advantage that the regression estimates indicate under what economic circumstances, i.e., for what values of the conditioning variables, intersection and spanning can or can not be rejected. Finally, in Section 2.4.3 we will discuss the use of conditioning variables as, e.g., in Shanken (1990) and Ferson & Schadt (1996). In that case variances and covariances are allowed to vary over time as well.

2.4.1 Incorporating conditional information using scaled returns

Suppose that z_t is a $(L-1)$-dimensional vector of instruments that has predictive power for R_{t+1} and r_{t+1}, and define the L-dimensional vector Z_t as $Z_t \equiv (1\ z_t')'$. A common way to use these instruments is to look at *scaled returns*: $Z_t \otimes R_{t+1}$. If M_{t+1} is a valid stochastic discount factor, then from (2.1) we have:

$$E[M_{t+1}(Z_t \otimes R_{t+1}) \mid I_t] = Z_t \otimes \iota_K.$$

Taking unconditional expectations, this yields

$$E[M_{t+1}(Z_t \otimes R_{t+1})] = E[Z_t \otimes \iota_K]. \tag{2.31}$$

Thus, the scaled return $Z_{i,t}R_{j,t+1}$ has an average price equal to $E[Z_{i,t}]$. The scaled returns can be interpreted as the payoffs of a strategy where each period an amount equal to $Z_{i,t}$ dollars is invested in a security, yielding a payoff equal to $Z_{i,t}R_{j,t+1}$. Therefore, we can also think of $Z_t \otimes R_{t+1}$ as the returns on *managed portfolios* (see, e.g., Cochrane, 1997). By allowing for such managed portfolios, we take into account that investors may use dynamic strategies, based on the realized values of Z_t. In effect this increases the set of available assets by a factor L.

To simplify notation, denote the $(L \times K)$-dimensional vector $Z_t \otimes R_{t+1}$ by R_{t+1}^Z. Also, denote the $(L \times K)$-dimensional vector $E[Z_t \otimes \iota_K]$ by q_K. For further reference, r_{t+1}^Z and q_N are defined in a completely analogous way. Valid stochastic discount factors M_{t+1}^Z now have to satisfy

$$E[M_{t+1}^Z R_{t+1}^Z] = q_K. \tag{2.32}$$

Following the same line of reasoning as in Sections 2.1 and 2.2, it is straightforward to show that the minimum variance stochastic discount factor with expectation v is given by

$$\begin{aligned} m_R^Z(v)_{t+1} &= v + \alpha^Z(v)'(R_{t+1}^Z - \mu_R^Z), \\ \alpha^Z(v) &= (\Sigma_{RR}^Z)^{-1}(q_K - v\mu_R^Z). \end{aligned} \tag{2.33}$$

This expression for the volatility bound is a straightforward generalization of the one given in (2.4) and (2.5). The restrictions imposed by the hypotheses of intersection and spanning also turn out to be very similar to the ones given in previous sections, as we will see below.

Thus, conditioning information can be incorporated by including managed portfolios, the returns of which depend on the conditioning variables. If there is to be conditional intersection or spanning of r_{t+1} by R_{t+1}, the unconditional volatility bound (or mean-variance frontier) of R_{t+1}^Z must intersect or span the volatility bound (or mean-variance frontier) of (R_{t+1}^Z, r_{t+1}^Z). The interest is in the returns R_{t+1} and r_{t+1} themselves *plus* the returns on all the managed portfolios. Intersection or spanning is equivalent to

$$E[r_{t+1}^Z m_R^Z(v)_{t+1}] = q_N, \tag{2.34}$$

for one value of v or for all values of v respectively. To see which restrictions these hypotheses imply, substitute (2.33) into (2.34) to obtain

$$(\mu_r^Z - \beta^Z \mu_R^Z)v + (\beta^Z q_K - q_N) = 0, \tag{2.35}$$

for intersection, and

$$(\mu_r^Z - \beta^Z \mu_R^Z) = 0, \text{ and } (\beta^Z q_K - q_N) = 0, \tag{2.36}$$

for spanning. Here β^Z is a $(L \times N) \times (L \times K)$ matrix with slope coefficients from a regression of r_{t+1}^Z on R_{t+1}^Z plus a constant. These restrictions are also given in Bekaert & Urias (1996).

The similarity with the case in which there was no conditioning information is obvious. The only difference in the restrictions is that in (2.35) and (2.36) we have $(\beta^Z q_K - q_N)$ instead of $(\beta\iota_K - \iota_N)$. The fact that q_K and q_N enter the restrictions reflects the fact that R_{t+1}^Z and r_{t+1}^Z are not really returns, in the sense that their current prices are not necessarily equal to one. The average prices of R_{t+1}^Z and r_{t+1}^Z are instead given by q_K and q_N. The average cost of the managed portfolios with payoff vector r^Z_{t+1} is given by the vector q_N, and the cost of the mimicking portfolios from R^Z_{t+1} is given by $\beta^Z q_K$. The interpretation of the restrictions given in Section 2.3.4 is therefore still valid.

The main disadvantage of this way of incorporating conditioning information is that the number of parameters to be estimated as well as the number of restrictions to be tested grows rapidly with the number of instruments L. The number of exogenous variables equals $K \times L$ and the number of restrictions to be tested equals $N \times L$ for the hypothesis of intersection, and $2N \times L$ for the hypothesis of spanning. This is the case because for each new instrument there are K new managed portfolios to be considered for the assets in R_{t+1} and N additional managed portfolios for the assets in r_{t+1}.

This problem can at least partially be circumvented if we are willing to assume a more specific form of predictability. Specifically, in the next section we make the assumption that only the expected returns of R_{t+1} and r_{t+1} depend linearly on the instruments z_t, whereas all variances and covariances are constants. In Section 2.4.3 the slope coefficients β are assumed to depend linearly on the instruments, which also allows for a straightforward way of incorporating conditional information in the regression framework to test for intersection and spanning.

2.4.2 Expected returns linear in the conditional variables

In this section we assume that there is a specific form of predictability, which allows us to incorporate conditioning information in a straightforward way in the regression framework for spanning and intersection. This way of incorporating conditioning information will be applied in the spanning tests for futures markets in Chapter 3. The assumption we make is that

$$\begin{aligned} E_t[R_{t+1}] &= \gamma_R' Z_t, \\ E_t[r_{t+1}] &= \gamma_r' Z_t, \end{aligned} \tag{2.37}$$

and the variances and covariances of R_{t+1} and r_{t+1} conditional on Z_t are given by $Var[R_{t+1} \mid Z_t] = \Omega_{RR}$, $Var[r_{t+1} \mid Z_t] = \Omega_{rr}$, and $Cov[r_{t+1}, R_{t+1} \mid Z_t] = \Omega_{rR}$. Starting from (2.1), the minimum variance stochastic discount factor is, in this particular setting, given by

$$\begin{aligned} m_R(v)_{t+1} &= v + \alpha(v)_t'(R_{t+1} - E_t[R_{t+1}]), \\ \alpha(v)_t &= \Omega_{RR}^{-1}(\iota_K - vE_t[R_{t+1}]). \end{aligned} \tag{2.38}$$

If there is intersection, $m_R(v)_{t+1}$ must price r_{t+1} correctly conditional on Z_t, which results in

$$\begin{aligned} \iota_N &= E_t[r_{t+1} m_R(v)_{t+1}] = v\gamma_r' Z_t + \Omega_{rR}\Omega_{RR}^{-1}(\iota_K - \gamma_R' Z_t) \\ &\Leftrightarrow (\gamma_r' - \Omega_{rR}\Omega_{RR}^{-1}\gamma_R') Z_t v + (\Omega_{rR}\Omega_{RR}^{-1}\iota_K - \iota_N) = 0. \end{aligned} \tag{2.39}$$

In case there is spanning this condition must again hold for every v, implying

$$(\gamma_r' - \Omega_{rR}\Omega_{RR}^{-1}\gamma_R')Z_t = 0 \text{ and } (\Omega_{rR}\Omega_{RR}^{-1}\iota_K - \iota_N) = 0. \tag{2.40}$$

It turns out that the regression framework that we used to test for spanning and intersection can easily be modified to test the restrictions in (2.39) and (2.40). In Appendix 2.A it is shown that in the regression

$$r_{t+1} = \gamma Z_t + \delta R_{t+1} + u_{t+1}, \tag{2.41}$$

with $E[u_{t+1}Z_t] = 0$, and $E[u_{t+1}R_{t+1}] = 0$, the OLS-estimates of γ and δ are consistent estimates of $(\gamma_r' - \Omega_{rR}\Omega_{RR}^{-1}\gamma_R')$ and $(\Omega_{rR}\Omega_{RR}^{-1}\iota_K - \iota_N)$ respectively, which are the parameters of interest in the restrictions in (2.39) and (2.40). The hypotheses of intersection and spanning can therefore be based on the OLS-estimates of (2.41). The hypothesis that there is intersection for a given value of v and Z_t can be tested by testing the restrictions

$$\gamma Z_t v + (\delta\iota_K - \iota_N) = 0, \tag{2.42}$$

and the hypothesis of spanning by testing the restrictions

$$\gamma Z_t = 0 \text{ and } (\delta\iota_K - \iota_N) = 0. \tag{2.43}$$

These restrictions are very similar to the restrictions implied by intersection and spanning in the unconditional case, except that the intercept α in (2.20) is replaced by γZ_t.

It can easily be seen from (2.42) and (2.43) that the number of restrictions to be tested for intersection and spanning is the same as in the unconditional case, which makes this method of incorporating conditional information somewhat more attractive than using scaled returns. Note that the hypotheses underlying (2.42) and (2.43) are that there is intersection or spanning for a particular value of Z_t, i.e., for a particular state of the economy. This has the additional advantage that the regression estimates of (2.41) allow us to make statements about the question in what states of the economy it will be useful to invest in r_{t+1} as well as in R_{t+1}. For instance, given the estimates of γ and δ in (2.41) and the concomitant covariance matrix, it is possible to derive confidence intervals for the values of Z_t for which there can be intersection or spanning.

If the hypothesis of interest is whether there is spanning regardless of the state of the economy, the restrictions in (2.43) should hold for all values of z_t, implying that each element of γ should be equal to 0. In that case, with L instruments and N assets in r_{t+1}, there are $L \times N$ restrictions to be tested, which, although smaller than the $2 \times L \times N$ restrictions in (2.36), can be a large number. Also, as follows readily from (2.42) and (2.43), in this case the hypothesis of intersection and the hypothesis of spanning both imply the same restrictions.

2.4.3 Slope coefficients β linear in the conditional variables

An alternative way of incorporating conditional information in the regression framework is suggested by Shanken (1990) and Ferson & Schadt (1996) e.g., where the slope coefficients β are assumed to be a linear function of the instruments. In the regression in (2.20),

$$r_{t+1} = \alpha + \beta R_{t+1} + \varepsilon_{t+1},$$

Shanken (1990) simply assumes that

$$\begin{aligned} \alpha &= a_0 + a_1 z_t, \\ \beta &= b_0 + b_1 z_t, \end{aligned} \tag{2.44}$$

where z_t are now supposed to be demeaned variables. Ferson & Schadt (1996) motivate (2.44) as a first order Taylor-series expansion for a general dependence of β on $Z_t = (1 \;\; z_t')'$. Let $Cov[r_{t+1}, R_{t+1} \mid Z_t] = \Sigma_{rR}(Z_t)$, and $Var[R_{t+1} \mid Z_t] = \Sigma_{RR}(Z_t)$, where $\Sigma(.)$ indicates some functional form for the covariance matrix. Starting from (2.13) intersection for a given zero-beta rate $\eta = 1/v$ conditional on Z_t means

$$\begin{aligned} E[r_{t+1} - \eta\iota_N] &= \beta(Z_t)E[R_{t+1} - \eta\iota_K] \Leftrightarrow \\ r_{t+1} - \eta\iota_N &= \beta(Z_t)(R_{t+1} - \eta\iota_K) + u_{t+1}, \end{aligned}$$

with $\beta(Z_t) = \Sigma_{rR}(Z_t)\Sigma_{RR}(Z_t)^{-1}$, $u_{t+1} \equiv (r_{t+1} - \beta(Z_t)R_{t+1}) - (E[r_{t+1}] - \beta(Z_t)E[R_{t+1}])$, and $E[u_{t+1} \mid Z_t] = 0$. Ferson & Schadt (1996) suggest a linear approximation of $\beta(Z_t)$:

$$\beta(Z_t) \approx b_0 + b_1 z_t, \tag{2.45}$$

from which

$$\begin{aligned} r_{t+1} &= a_0 + a_1 z_t + b_0 R_{t+1} + b_1 z_t R_{t+1} + \varepsilon_{t+1}, \\ a_0 &= \eta(\iota_N - b_0\iota_K), \\ a_1 &= -\eta b_1 \iota_K, \end{aligned} \tag{2.46}$$

with $\varepsilon_{t+1} = u_{t+1} + (\beta(Z_t) - b_0 - b_1 z_t)(R_{t+1} - \eta\iota_K)$, for which it is assumed that $E[\varepsilon_{t+1} \mid Z_t] = 0$. This gives precisely the regression in (2.20) where the regression parameters are linear in the instruments as assumed by Shanken (1990).

Intersection for a given value of $\eta = 1/v$ and z_t can now be tested by testing the restrictions that

$$(a_0 + a_1 z_t) + \{(b_0 + b_1 z_t)\iota_K - \iota_N\}\eta = 0. \tag{2.47}$$

As in the previous section, these restrictions give the additional advantage that statements can be made as in which state of the economy, i.e., for which values of z_t there is intersection. If there is intersection for all values of z_t, this implies

$$\begin{aligned} a_0 + (b_0\iota_K - \iota_N)\eta &= 0, \\ a_1 + b_1\iota_K\eta &= 0. \end{aligned}$$

Spanning for a given value of z_t is equivalent to

$$\begin{aligned} a_0 + a_1 z_t &= 0, \\ (b_0 + b_1 z_t)\iota_K &= \iota_N. \end{aligned} \tag{2.48}$$

Again, for a specific value of z_t, i.e., for specific economic conditions, these restrictions can easily be tested in the regression framework outlined above. If there is to be spanning under all economic conditions the restrictions are

$$\begin{aligned} a_0 &= 0, \\ b_0\iota_K &= \iota_N, \\ a_1 &= 0, \\ b_1 &= 0. \end{aligned}$$

If there are L instruments (including a constant) with K benchmark assets and N new assets, we now have $(K+1) \times N \times L$ restrictions to test, which is even larger than with the scaled returns in Section 2.4.1. Also, the numbers of parameters to be estimated is $(K+1) \times N \times L$. Thus, in terms of the number of parameters and the number of restrictions, this approach does not offer additional benefits over the use of scaled returns. However, this approach does have the benefit that it shows under what economic circumstances there may or may not be intersection or spanning.

Notice that this way of incorporating conditional information is very similar to the one suggested in the previous section. The restrictions on the regression parameters in (2.46) are analogous to the ones on the parameters in (2.41). The main difference arises because the slope coefficients for R_{t+1} also depend on the instruments, implying that the interaction term $z_t R_{t+1}$ should also be included in the regression. It is easy to see that the approach in the previous section can be interpreted as a special case of the approach outlined here, where only the intercepts in (2.20) are a function of the instruments z_t, whereas the slope coefficients are constant.

Summarizing, we have shown that a number of approaches is available to incorporate conditioning information in tests for intersection and spanning. Using either scaled returns or regression coefficients that are linear functions of the instruments, the regression approach outlined in Section 2.3 can easily be extended to test for intersection or spanning. The restrictions implied by the hypotheses of intersection and spanning are very similar to the case where there is no conditioning information (i.e., where the only instrument is a constant) and have very similar interpretations as well.

2.5 The relation between spanning tests, performance evaluation and optimal portfolio weights

So far the focus has been on the restrictions that are implied by the hypotheses of intersection and spanning on the distribution of R_{t+1} and r_{t+1} and on how these hypotheses can be tested. In this section the interest will be on the deviations from the restrictions. We will show that the test statistics and regression estimates have clear interpretations in terms of performance measures like Jensen's alpha and the Sharpe ratio as well as in terms of the new optimal portfolio weights. Since it is natural to think about these performance measures in terms of mean-variance efficient portfolios, most of the analysis in this section will be in terms of mean-variance frontiers rather than volatility bounds. Nonetheless, the duality between these two frontiers also holds for these performance measures. Interpretations of tests for mean-variance efficiency, intersection, and spanning in terms of performance measures can also be found in Gibbons, Ross & Shanken (1989), Jobson & Korkie (1982, 1984, 1989), and Kandel & Stambaugh (1989).

2.5.1 Performance measures

To set the stage, define the vector of *Jensen's alphas*, or *Jensen performance measures*, $\alpha_J(\eta)$, as the intercepts in a regression of the N excess returns $(r_{t+1} - \eta\iota_N)$ on the excess returns of the K benchmark assets, $(R_{t+1} - \eta\iota_K)$:

$$r_{t+1} - \eta\iota_N = \alpha_J(\eta) + \beta(R_{t+1} - \eta\iota_K) + \varepsilon_{t+1}, \tag{2.49}$$

with $E[\varepsilon_{t+1}] = E[\varepsilon_{t+1}R_{t+1}] = 0$. Since it is not assumed that there exists a risk free asset, we define excess returns as the return on an asset or portfolio in excess of a given zero-beta rate η. Alternatively, when regressing r_{t+1} on R_{t+1} as in (2.20), it follows that Jensen's alpha is equal to

$$\alpha_J(\eta) = \alpha + (\beta\iota_K - \iota_N)\eta, \tag{2.50}$$

where $\alpha = \mu_r - \beta\mu_R$ and $\beta = \Sigma_{rR}\Sigma_{RR}^{-1}$. Notice from this expression that the hypothesis that there is intersection for a given value of η is equivalent to the hypothesis that the Jensen performance measure is zero, i.e., $\alpha_J(\eta) = 0$. Similarly, the hypothesis of spanning is equivalent to the hypothesis that $\alpha_J(\eta) = 0$, $\forall\eta$. Recall from Section 2.3.3, that the regression in (2.49) produces the same intercept $\alpha_J(\eta)$ as a regression of $r_{t+1} - \eta\iota_N$ on the excess return of a portfolio w_R^* that is mean-variance efficient for $R_{t+1}$and that has η as its zero beta rate, i.e.,

$$r_{t+1} - \eta\iota_N = \alpha_J(\eta) + \beta^*(R_{t+1}^* - \eta) + \varepsilon_{t+1}.$$

It is common in the literature to define Jensen's alpha as the intercept of a regression of r_{t+1} in excess of the risk free rate on the return of the market portfolio in excess of the risk free rate. The definition in (2.49) is more general and has this more traditional definition as a special case if there exists a risk free asset ($\eta = R_t^f$) and if the market portfolio is mean-variance efficient ($R_{t+1}^* = R_{t+1}^m$). The Jensen measure in (2.49) is also referred to as the *generalized* Jensen measure. Given the minimum variance stochastic discount factor $m_R(v)_{t+1}$ as defined in (2.4) and (2.5), it can easily be seen that the generalized Jensen measure is also equal to $\lambda(v)/v$ as defined in (2.28).

The *Sharpe ratio* of a portfolio with return R_{t+1}^p is defined as the expected excess portfolio return, divided by the standard deviation of portfolio return,

$$Sh(R_{t+1}^p, \eta) \equiv \frac{E[R_{t+1}^p] - \eta}{\sigma(R_{t+1}^p)}.$$

By definition, for a given expected portfolio return, or for a given standard deviation of portfolio return, the maximum attainable (absolute) Sharpe ratio is the Sharpe ratio of the minimum-variance efficient portfolio. For a minimum-variance efficient portfolio w_R^* of the K assets R_{t+1} with zero-beta rate η, the Sharpe ratio is equal to the slope of the line tangent to the frontier originating at $(0, \eta)$ in mean-standard deviation space, and is denoted by $\theta_R(\eta)$:

$$\theta_R(\eta) = \frac{E[R_{t+1}^*] - \eta}{\sigma(R_{t+1}^*)}, \tag{2.51}$$

where $R_{t+1}^* \equiv w^{*\prime} R_{t+1}$.

Although both Jensen's alpha and the Sharpe ratio are used as performance measures, there is an important difference between the two. Whereas the Sharpe ratio is defined in terms of the characteristics of one portfolio (the expected excess portfolio return and its standard deviation), Jensen's alpha is defined in terms of one asset or portfolio relative to another. Sharpe ratios answer the question whether one portfolio is to be preferred over another, whereas Jensen's alpha answers the question whether investors can improve the efficiency of their portfolio by investing in the asset. However, there is a close relation between the two measures, in that Jensen's alphas together with the covariance matrix of the error terms ε_{t+1} in (2.20) (and (2.49)) determine the potential improvement in the maximum attainable Sharpe ratio from adding the new assets r_{t+1}. Recall from Section 2.2.2 that we defined the variables $A \equiv \iota'\Sigma^{-1}\iota$, $B \equiv \mu'\Sigma^{-1}\iota$, and $C \equiv \mu'\Sigma^{-1}\mu$. For the set R_{t+1} these variables will be denoted as A_R, B_R, and C_R, whereas the absence of subscripts implies that these variables refer to the larger set (R_{t+1}, r_{t+1}). Using partitioned inverses, notice that

$$\Sigma^{-1} = \begin{pmatrix} \Sigma_{RR} & \Sigma_{Rr} \\ \Sigma_{rR} & \Sigma_{rr} \end{pmatrix}^{-1} = \begin{pmatrix} \Sigma_{RR}^{-1} + \beta'\Sigma_{\varepsilon\varepsilon}^{-1}\beta & -\beta'\Sigma_{\varepsilon\varepsilon}^{-1} \\ -\Sigma_{\varepsilon\varepsilon}^{-1}\beta & \Sigma_{\varepsilon\varepsilon}^{-1} \end{pmatrix}. \tag{2.52}$$

From this it follows that

$$\begin{aligned} A &= \iota_K'\Sigma_{RR}^{-1}\iota_K + \iota_K'\beta'\Sigma_{\varepsilon\varepsilon}^{-1}\beta\iota_K - 2\iota_K'\beta'\Sigma_{\varepsilon\varepsilon}^{-1}\iota_N + \iota_N'\Sigma_{\varepsilon\varepsilon}^{-1}\iota_N \\ &= A_R + (\beta\iota_K - \iota_N)'\Sigma_{\varepsilon\varepsilon}^{-1}(\beta\iota_K - \iota_N), \end{aligned} \tag{2.53}$$

where $\beta = \Sigma_{rR}\Sigma_{RR}^{-1}$ and $\Sigma_{\varepsilon\varepsilon}$ is the covariance matrix of ε_{t+1}, the error term in the regression in (2.20). In a similar way it can easily be shown that

$$B = B_R + \alpha'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K), \tag{2.54a}$$

$$C = C_R + \alpha'\Sigma_{\varepsilon\varepsilon}^{-1}\alpha, \tag{2.54b}$$

where $\alpha = \mu_r - \beta\mu_R$, the intercept in the regression in (2.20).

It is easy to show that for a given η, the Sharpe ratio of a mean-variance efficient portfolio w_R^* can be written as

$$\theta_R(\eta) = (C_R - 2B_R\eta + A_R\eta^2)^{1/2}. \tag{2.55}$$

A similar expression holds of course for $\theta(\eta)$, the maximum attainable Sharpe ratio of the larger set (R_{t+1}, r_{t+1}). Combined with (2.53) and (2.54) this gives for the squared Sharpe ratio

$$\begin{aligned} \theta(\eta)^2 &= C - 2B\eta + A\eta^2 \\ &= (C_R - 2B_R\eta + A_R\eta^2) \\ &\quad + (\alpha'\Sigma_{\varepsilon\varepsilon}^{-1}\alpha - 2\alpha'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K)\eta + (\iota_N - \beta\iota_K)'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K)\eta^2) \\ &= \theta_R(\eta)^2 + \alpha_J(\eta)'\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(\eta). \end{aligned} \tag{2.56}$$

Thus, the change in maximum attainable squared Sharpe ratios equals the inner product of the vector of Jensen's alphas, $\alpha_J(\eta)$, weighted by the inverse of the covariance matrix of ε_{t+1}.[4] If there is only one new asset, $N = 1$, the term $\alpha_J(\eta)/\sigma(\varepsilon)$ is known as the *adjusted* Jensen measure. Notice once more that $\theta_R(\eta)$ and $\theta(\eta)$ characterize portfolios of R_{t+1} and $(R_{t+1}', r_{t+1}')'$, respectively, whereas $\alpha_J(\eta)$ and $\Sigma_{\varepsilon\varepsilon}$ follow from a regression of r_{t+1} on R_{t+1}, and measure the performance of r_{t+1} relative to R_{t+1}. Stated differently, whereas Sharpe ratios can be used to compare the performance of different portfolios, Jensen's alpha gives the potential improvement in performance when the additional assets are included in the portfolio. The hypotheses of

[4] This result can also be found in Jobson & Korkie (1984) for instance.

intersection and spanning imply that Jensen's alpha, $\alpha_J(\eta)$, is zero for one or for all values of η respectively. Therefore, if there is intersection (spanning) then there is no improvement in the Sharpe measure possible by including the additional assets r_{t+1} in the investors portfolio.

2.5.2 Changes in optimal portfolio weights

The performance measures and the intersection regressions discussed above can also be used to infer the changes in optimal portfolio holdings when adding the assets r_{t+1}. In this section we will show that given the initial mean-variance efficient portfolio of the benchmark assets and the OLS-estimates of the regression parameters in (2.20), it is straightforward to determine the new optimal portfolio weights. In order to do so, consider the mean-variance efficient portfolio for the extended set (R_{t+1}, r_{t+1}) for a given value of η:

$$w^* = \gamma^{-1}\Sigma^{-1}(\mu - \eta\iota).$$

Substituting the partitioned inverse as given in (2.52) in the expression for w^* gives that the optimal portfolio weights for the new assets, w_r^*, can be written as

$$\begin{aligned} w_r^* &= \gamma^{-1}\Sigma_{\varepsilon\varepsilon}^{-1}((\mu_r - \beta\mu_R) - (\iota_N - \beta\iota_K)\eta) \\ &= \gamma^{-1}\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(\eta). \end{aligned} \tag{2.57}$$

Thus, the optimal portfolio weights w_r^* are determined by the vector of Jensen's alphas and the covariance matrix of the residuals of the OLS-regression of r_{t+1} on R_{t+1}.[5]

In deriving the new optimal portfolio weights, a problem in (2.57) is that the coefficient of risk aversion γ is present. Notice that this is a different coefficient than the one that appears in the optimal portfolio $\widetilde{w}_R^*$ of the smaller set R_{t+1}:

$$\widetilde{w}_R^* = \widetilde{\gamma}_R^{-1}\Sigma_{RR}^{-1}(\mu_R - \eta\iota_K),$$

where we now also add a ˜ to indicate that a variable refers to the set of benchmark assets R_{t+1} only. It is only the zero-beta return η that is the same in both problems, since we test whether there is intersection for a fixed value of η. Similarly, the expected returns on the portfolios $\widetilde{w}_R^*$ and w^* are different, and we indicate these with $\widetilde{m}_R$ and m respectively, i.e., $\widetilde{m}_R \equiv \widetilde{w}_R^{*\prime}\mu_R$, and $m \equiv w^{*\prime}\mu$. In order to substitute out the risk aversion parameter γ, note that

$$\begin{aligned} \gamma &= B - \eta A = B_R - \eta A_R + \alpha_J(\eta)'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K) \\ &= \widetilde{\gamma}_R + \alpha_J(\eta)'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K), \end{aligned}$$

[5] As an aside, in terms of volatility bounds, notice that $w_r^*\gamma = -\alpha_r(1/\eta)$, i.e., the elements of $\alpha(v)$ in (2.5) that correspond to r_{t+1}. Thus if we want to know the minimum variance stochastic discount factor from (R_{t+1}, r_{t+1}), rather than from R_{t+1}, the projection coefficients corresponding to the additional assets r_{t+1} are given by $-\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(\eta)$.

and that

$$\widetilde{\gamma}_R = \frac{\widetilde{m}_R - \eta}{\widetilde{w}_R^{*\prime}\Sigma_{RR}\widetilde{w}_R^*} = \frac{\theta_R(\eta)^2}{\widetilde{m}_R - \eta}.$$

Using these latter two expressions, the optimal portfolio weights w_r^* can be expressed as

$$w_r^* = \left(\frac{\widetilde{m}_R - \eta}{\theta_R(\eta)^2 + (\widetilde{m}_R - \eta)\alpha_J(\eta)'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K)}\right)\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(\eta). \tag{2.58}$$

The interesting thing about (2.58) is that it contains only variables that either result from the initial optimal portfolio $\widetilde{w}_R^*$, or from a regression of r_{t+1} on R_{t+1}.

Along the same lines it is straightforward to show that the new optimal weights w_R^* are given by

$$w_R^* = \left(\frac{\theta_R(\eta)^2}{\theta_R(\eta)^2 + (\widetilde{m}_R - \eta)\alpha_J(\eta)'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K)}\right)\widetilde{w}_R^* - \beta' w_r^*. \tag{2.59}$$

Again, this expression only depends on characteristics of the old portfolio, $\widetilde{w}_R^*$, and the regression output. Therefore, given the initial mean-variance efficient portfolio $\widetilde{w}_R^*$ of the benchmark assets and the OLS-estimates of the regression in (2.20), (2.58) and (2.59) answer the question how to adjust the portfolio in order to obtain the new mean-variance efficient portfolio w^*.

In order to give an interpretation of the new portfolio weights in (2.58) and (2.59), it is useful to rewrite them in the following way:[6]

$$w_r^* = \frac{m - \eta}{\theta(\eta)^2}\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(\eta), \tag{2.60}$$

and

$$w_R^* = \frac{\theta_R(\eta)^2}{\theta(\eta)^2}\frac{m - \eta}{\widetilde{m}_R - \eta}\widetilde{w}_R - \beta' w_r^*. \tag{2.61}$$

If there is only one new asset, i.e., $N = 1$, Equation (2.60) first of all shows that $\alpha_J(\eta)$ determines the sign of the new portfolio weight w_r^* (given that $m - \eta > 0$): if Jensen's alpha is positive (negative) the investor can improve the performance of his portfolio by taking long (short) positions in the new asset. When there is more than one new asset, the sign of the portfolio weights is not only determined by the sign of Jensen's alpha, but also by the inverse of the covariance matrix of ε_{t+1}. If the mean-variance frontier is not strongly affected by the introduction of the new assets, then $(\theta_R(\eta)^2/\theta(\eta)^2)(m-\eta)/(\widetilde{m}_R-\eta) \approx 1$, and the coefficients β show which of the old assets are replaced by the new assets. These results will be illustrated with a numerical example in Section 2.5.4.

Finally, notice that we did not consider a risk free asset. The portfolio weights given above are for the tangency portfolio when the zero-beta rate is η. If a risk free asset is available, we can

[6] Here we use the fact that $\theta_R(\eta)^2/(\widetilde{m}_R - \eta) = A_R - \eta B_R$, and that $A_R - \eta B_R + \alpha_J(\eta)'\Sigma_{\varepsilon\varepsilon}^{-1}(\iota_N - \beta\iota_K) = A - \eta B$.

replace η with R^f in (2.60) and (2.61) and these equations still give the portfolio weights for the tangency portfolio. The new tangency portfolio has an expected return equal to m, whereas the old tangency portfolio has an expected return $\widetilde{m}_R$. Notice though, that in case a risk free asset is available it is easy to shift funds between the tangency portfolio and the risk free asset and to let the expected portfolio return vary. For practical purposes, the interest may be in the new portfolio w^* that has the same expected return as the old portfolio. Given that there is a risk free asset available, this is easily achieved by letting $m - R^f = \widetilde{m}_R - R^f$. In this case Equations (2.60) and (2.61) simplify to

$$w_r^* = \frac{m - R^f}{\theta^2} \Sigma_{\varepsilon\varepsilon}^{-1} \alpha_J \tag{2.62}$$

and

$$w_R^* = \frac{\theta_R^2}{\theta^2} \widetilde{w}_R - \beta' w_r^*. \tag{2.63}$$

Notice that here it is not necessarily the case that the weights in w_r^* and w_R^* sum to one. The investor will have to borrow or lend a fraction $(1 - \iota_K' w_R^* - \iota_N' w_r^*)$ to achieve an expected portfolio return equal to m.

2.5.3 Interpretation of spanning and intersection tests in terms of performance measures

Finally, we want to relate the Wald test-statistics presented in Section 2.3 to the performance measures discussed above. It will be shown that these test-statistics can be expressed as changes in maximum Sharpe ratios of R_{t+1} and (R_{t+1}, r_{t+1}) respectively. Therefore, they have a clear economic interpretation. In order to interpret the test-statistics for intersection and spanning in terms of performance measures, recall the basic regression model in (2.20):

$$r_{t+1} = \alpha + \beta R_{t+1} + \varepsilon_{t+1},$$

where intersection for a given value of η means that

$$\alpha_J(\eta) = \alpha + (\beta\iota_K - \iota_N)\eta = 0.$$

Thus, the restrictions on the regression coefficients that are imposed by the hypothesis of intersection have a natural interpretation in terms of Jensen's alphas, and - as noted before - testing whether there is intersection for η, is equivalent to testing whether Jensen's alpha is zero. Testing for spanning is of course equivalent to testing whether the Jensen's alphas are zero for all values of η.

It can be shown that the test statistics for intersection and spanning, ξ_W^{int} and ξ_W^{span}, presented in Section 2.3.4, can also be interpreted in terms of Jensen's alphas and Sharpe ratios. To see this, start again from the specification of the regression equation in (2.23):

$$r_{t+1} = (I_N \otimes \begin{pmatrix} 1 & R'_{t+1} \end{pmatrix})\, b + \varepsilon_{t+1}.$$

Note that (using partitioned inverses) the asymptotic covariance matrix of the OLS-estimates of b, $\widehat{b}$ in (2.23) is given by

$$\begin{aligned} & \Sigma_{\varepsilon\varepsilon} \otimes \begin{pmatrix} 1 & \mu'_R \\ \mu_R & E[R_t R'_t] \end{pmatrix}^{-1} \\ = \; & \Sigma_{\varepsilon\varepsilon} \otimes \begin{pmatrix} 1 + \mu'_R \Sigma_{RR}^{-1} \mu_R & -\mu'_R \Sigma_{RR}^{-1} \\ -\Sigma_{RR}^{-1} \mu_R & \Sigma_{RR}^{-1} \end{pmatrix}. \end{aligned} \tag{2.64}$$

Straightforward algebra shows that premultiplying (2.64) with $H(\eta)_{int}$ and postmultiplying with $H(\eta)'_{int}$ as defined in (2.25), yields

$$Var[\widehat{\alpha}_J(\eta)] = \Sigma_{\varepsilon\varepsilon}(1 + \theta_R(\eta)^2), \tag{2.65}$$

where the Sharpe ratio $\theta_R(\eta)$ was defined in (2.55). Since from the analysis above we know that the term $h(\eta)_{int}$ as defined in (2.25) equals $\alpha_J(\eta)$, (2.56) can be used to rewrite the test statistic for intersection, ξ_W^{int}, as

$$\xi_W^{int} = T \frac{\widehat{\alpha}_J(\eta)' \widehat{\Sigma}_{\varepsilon\varepsilon}^{-1} \widehat{\alpha}_J(\eta)}{1 + \widehat{\theta}_R(\eta)^2} = T \left(\frac{1 + \widehat{\theta}(\eta)^2}{1 + \widehat{\theta}_R(\eta)^2} - 1 \right), \tag{2.66}$$

where $\widehat{\theta}_R(\eta)$, $\widehat{\theta}(\eta)$, and $\widehat{\alpha}_J(\eta)$ are the sample Sharpe ratios and Jensen's alpha respectively. Equation (2.66) is a well known result from, e.g., Jobson & Korkie (1982) and Gibbons, Ross & Shanken (1989). It clearly shows that the Wald test statistic for intersection can easily be interpreted as the percentage increase in squared Sharpe ratios scaled by the sample size. Under the null-hypothesis that there is intersection, $\theta(\eta) = \theta_R(\eta)$ and the increase of the sample Sharpe ratios scaled by the sample size T (as in (2.66)) will asymptotically have a $\chi^2_{(N)}$-distribution. Gibbons, Ross & Shanken (1989) study the small sample properties of this test statistic in case there is a risk free asset, as well as the distribution under the alternative hypothesis. Kandel & Stambaugh (1987) and Shanken (1987) extend their results to the case where the mean-variance efficient benchmark portfolio (or intersection portfolio) can not be observed but has a given correlation with observed proxy portfolios.

For the spanning test-statistic, a similar interpretation can be given. Let η_R^0 denote the expected return on the global minimum variance portfolio of R_{t+1}, i.e., $\eta_R^0 = B_R/A_R$, and let the variance of this portfolio be given by $(\sigma_R^0)^2$. Similarly, let $(\sigma^0)^2$ be the global minimum variance of

(R_{t+1}, r_{t+1}). It is shown in Appendix 2.B that the Wald test-statistic for spanning, ξ_W^{span}, can be written as

$$\xi_W^{span} = T\left(\frac{1+\widehat{\theta}(\widehat{\eta}_R^0)^2}{1+\widehat{\theta}_R(\widehat{\eta}_R^0)^2} - 1\right) + T\left(\frac{(\widehat{\sigma}_R^0)^2}{(\widehat{\sigma}^0)^2} - 1\right). \tag{2.67}$$

This shows that the spanning test-statistic consists of two parts. The first part is similar to the test-statistic for intersection in (2.66) and is determined by a change in Sharpe ratios. The Sharpe ratios in (2.67) are for a zero-beta rate equal to the (in-sample) expected return on the global minimum variance portfolio however, and therefore are the slopes of the asymptotes of the mean-variance frontier. Notice that the slope of the upper limb of the frontier is simply the negative of the slope of the lower limb of the frontier, and therefore, the squared Sharpe ratios for those two extremes are the same. The first term of the spanning test-statistic in a sense measures whether there is intersection at the most extreme points of the frontier (i.e., whether there is a limiting form of intersection if we go sufficiently far up or down the frontier). The second term of the statistic in (2.67) is determined by the change in the global minimum variance of the portfolios, and measures whether the point most to the left on the frontier changes or not. Put differently, the first term measures whether there is intersection for a mean-variance investor with a very small risk aversion ($\gamma = 0$), while the second term measures whether there is intersection for a mean-variance investor with a very high risk aversion ($\gamma = \infty$). Note that in the second term the old global minimum variance appears in the numerator and the new global minimum variance in the denominator, since this variance can only decrease as assets are added to the portfolio. Therefore, both terms in (2.67) are always larger than or equal to one. Jobson & Korkie (1989) derive a similar result for a likelihood ratio test for spanning.

2.5.4 The empirical example continued

To illustrate the relation between testing for intersection, performance measures and optimal portfolio weights, we will continue the empirical example of Section 2.3.5. Recall that the benchmark indices are the MSCI Indices for the US and Canada and the interest is in whether the efficient set can be improved by including the indices for the Netherlands and/or Japan in the portfolio. As in Section 2.3.5 we consider the case where the zero-beta rate $\eta = 1.0025$, which corresponds to an annual rate of approximately 3%. Table 2.3 shows the relevant data for the example.

Using the summary statistics in Table 2.1, if the zero-beta rate is 0.25%, the optimal portfolio of the benchmark assets has an expected return of 1.62% per month and a Sharpe ratio of 0.28, implying that the standard deviation of the portfolio return is 5.67%. This efficient portfolio

Table 2.3: The relation between intersection tests, performance measures and optimal portfolio weights.

The first part of the table shows the Sharpe ratios and the expected returns of mean-variance efficient portfolios of the benchmark assets (=MSCI Indices of the US and Canada), the benchmark assets plus the index of the Netherlands, and the benchmark assets plus the Japanese index respectively, when the zero-beta rate $\eta = 1.0025$. The second part reports the results of an OLS-regression of the returns of the Dutch and Japanese indices on the returns of the benchmark indices and a constant, as well as the implied Jensen measure when the zero-beta rate is 1.0025. Finally, the last part shows the optimal mean-variance portfolio weights of the assets when the zero beta rate is 1.0025. All results are based on monthly unhedged US dollar-based returns for the period from January 1985 until June 1996.

	US+Can	+NL	+Jap
$\theta(1.0025)$	0.285	0.396	0.308
m (%)	1.62	2.14	1.64
Regression estimates			
Const.		0.29	0.54
US		0.41	0.09
Can		0.31	0.39
$\sigma(\varepsilon)$ (%)		3.35	7.26
$\alpha_J(1.0025)$ (%)		0.92	0.85
Portfolio weights			
US	1.46	0.65	1.26
Can	-0.46	-0.64	-0.49
NL		0.99	
Jap			0.24

consists of an investment of 146% in the US index and a short position of 46% in the Canadian index. Given the summary statistics in Table 2.1 it is not surprising that investors want to sell the Canadian index short: the mean return for the US index is much higher than for the Canadian index, whereas the standard deviation of the US returns are smaller than for Canada and the two indices have a rather high correlation.

The second part of Table 2.2 gives the results from an OLS-regression of the returns on the Dutch and Japanese indices on the US and the Canadian indices respectively. From the reported regression coefficients Jensen's alpha can be calculated as $\alpha_J = \alpha + (\beta\iota_2 - 1)\eta$, which is 0.92% for the Netherlands and 0.85% for Japan. Thus, if the investor would include the Netherlands or Japan in his portfolio, it would be optimal to take long positions in these countries. The exact portfolio weights can be determined using Equation (2.58), from which we find for instance

$$w^*_{NL} = \frac{1.37\%}{0.285^2 + 1.37\%(0.92\% * 0.28/3.35\%^2)} \times \frac{0.92\%}{3.35\%^2} = 0.99.$$

If the investor wants to invest 99% of his portfolio in the Netherlands, he will have to sell part of his old portfolio. The new portfolio holdings in the benchmark assets can be found using Equation

(2.59), from which

$$\begin{aligned} w_R^* &= \frac{0.285^2}{0.285^2 + 1.37\%(0.92\% * 0.28/3.35\%^2)}\widetilde{w}_R^* - \beta' 0.99 \\ &= 0.72\widetilde{w}_R^* - 0.99\beta'. \end{aligned}$$

Thus, the investor will only hold a fraction 0.72 of his old portfolio, implying that 38% of his funds will become available for investment in the new asset, and furthermore he will sell $0.99\beta'$ of the benchmark assets. Since the slope coefficients of the benchmark assets are 0.41 for the US index and 0.31 for the Canadian index, most of the additional funds that are needed for buying the Dutch index come from selling his holdings in the US-index. The final results for the optimal portfolio holdings can be found in the bottom part of Table 2.3, which also shows the results for Japan.

The first two rows of Table 2.3 also quantify the benefits from including the Netherlands or Japan in the investors' portfolio. For instance, when including the Netherlands, with a zero-beta rate of 0.25% the Sharpe ratio increases from 0.285 to 0.396, which is very large in economic terms. Thus, including the Netherlands in the investment portfolio causes a shift in the mean-variance frontier that is at least significant in economic terms. Given that the results in Table 2.3 are based on 138 observations, the Wald test-statistic for intersection can be calculated using (2.66):

$$\xi_W^{int} = 138 \times \left(\frac{1+0.396^2}{1+0.285^2} - 1\right) = 9.64.$$

Since the asymptotic distribution of this test-statistic is χ_1^2, the associated p-value is 0.002, which shows that the shift in the mean-variance frontier is also significant in statistical terms, as we already showed in Section 2.3.5. As a final point, notice that the portfolios in Table 2.3 all require short positions in the Canadian index. In Chapter 4 it will be shown how we can test for intersection and spanning if short sales of the indices are not allowed.

2.6 Specification error bounds and intersection

As in the previous section, in this section the focus will be on deviations from intersection rather than on intersection itself. In a recent paper Hansen & Jagannathan (1997) analyze *specification errors* in stochastic discount factor models which, in some special cases, can be interpreted as deviations from intersection. They derive bounds on the magnitude of these specification errors which we will apply to models for futures risk premia in Chapter 6. Therefore, the analysis in this section also serves as an introduction to Chapter 6. Recall from the discussion in Section 2.2.1 that each asset pricing models assigns a particular function to the pricing kernel M_{t+1}. Hansen & Jagannathan (1997) note that the pricing kernels implied by most asset pricing models do not yield correct asset prices, either because the asset pricing model can only be viewed as an approximation, or because of measurement error. Measurement errors are for instance often considered to be an important problem in measuring consumption and testing consumption based asset pricing models. Therefore, the pricing kernel implied by an asset pricing model will in general only serve as a *proxy* stochastic discount factor, that will not yield the correct prices or expected payoffs of the assets under consideration. The interest of Hansen & Jagannathan is in the least squares distance between such a proxy stochastic discount factor and the set of valid stochastic discount factors. They derive a lower bound on this distance, the *specification error bound,* as a measure of how well the model performs. These specification error bounds will be derived formally below and it will also be shown that these bounds have a clear economic interpretation in terms of maximum pricing errors or maximum expected payoff errors implied by the asset pricing model. Hansen, Heaton, & Luttmer (1995) derive the limiting distribution for the corresponding estimator of the specification error bounds.

It turns out to be the case that if we take the minimum variance stochastic discount factor for the subset R_{t+1} as a proxy stochastic discount factor for the larger set of assets (R_{t+1}, r_{t+1}), we can interpret the specification error bounds in terms of mean-variance intersection and the performance measures discussed in the previous section. In particular, provided that both the proxy stochastic discount factor and the discount factors that price R_{t+1} and r_{t+1} correctly have the same expectation v, the squared specification error bound scaled by v turns out to be equal to the difference between the maximum squared Sharpe ratio implied by the set R_{t+1} and the maximum squared Sharpe ratio implied by (R_{t+1}, r_{t+1}). This also allows us to interpret the specification errors in terms of mean-variance portfolio choice again. Given that a mean-variance investor is aware of the fact that a portfolio chosen from the subset R_{t+1} is suboptimal relative to a portfolio chosen from the larger set (R_{t+1}, r_{t+1}), the specification error bound gives an estimate of the extent to which the portfolio is suboptimal in terms of Sharpe ratios.

2.6.1 Specification error bounds

As noted above, in Hansen & Jagannathan (1997) the interest is in *proxy* stochastic discount factors, denoted by y_{t+1}, that assign approximate prices to portfolio payoffs. For instance, the CAPM implies that the proxy is of the form $a + bR^m_{t+1}$, with R^m_{t+1} the return on the market portfolio, and the model in Chapter 6 implies a proxy of the form $a + bR^m_{t+1} + c'q^m_t$, where q^m_t is a vector of net market exposures in several nontraded assets, i.e., hedging pressures. As before, let R^p_{t+1} be the return on some portfolio, not necessarily mean-variance efficient, such that $w^{p\prime}\iota_K = 1$. The expected price assigned to such a portfolio by a proxy stochastic discount factor will be denoted by $\pi^a(R^p_{t+1})$:

$$E[y_{t+1}R^p_{t+1}] = \pi^a(R^p_{t+1}). \tag{2.68}$$

Of course, valid stochastic discount factors M_{t+1} would assign a price $\pi(R^p_{t+1}) = 1$ to any portfolio w^p that satisfies $w^{p\prime}\iota_K = 1$. Because the proxy y_{t+1} may be derived from an asset pricing model that is strictly speaking not valid, or because the proxy may be measured with error, the prices assigned by the proxy, $\pi^a(R^p_{t+1})$, will in general not be equal to one. The discussion here is somewhat restrictive because we only consider payoffs that are returns, i.e., payoffs with (correct) prices equal to one. Hansen & Jagannathan (1997) take more general payoffs x_{t+1} with current prices q_t. Given that we want to establish the relation between specification errors and mean-variance intersection, the use of returns is not very restrictive however. Moreover, the results derived below can easily be adjusted to the results of Hansen & Jagannathan along the lines of Section 2.4.1, where we incorporated conditioning information by allowing for payoffs $z_t \otimes R_{t+1}$ with current prices q_t.

A second way in which the results here are somewhat more restrictive than the ones in Hansen & Jagannathan (1997) is that we will always consider valid stochastic discount factors $M(v)_{t+1}$ that have the same expectation as the proxy y_{t+1}, i.e., $v = E[y_{t+1}]$. This may be considered as restrictive, since this assumption in fact requires that the proxy assigns the correct price to the risk free payoff, if it exists. Once more, given that the interest here is in the relation with mean-variance intersection in the absence of a risk free asset, and given that we always defined intersection for a known value of v, this is not restrictive for our purposes.

The problem addressed in Hansen & Jagannathan (1997) is to derive a lower bound δ on the distance between y_{t+1} and the set of stochastic discount factors that price R_{t+1} correctly, which we denote as $\mathcal{M}$:

$$\delta = \min_{\{M_R(v)_{t+1} \in \mathcal{M}\}} \| y_{t+1} - M_R(v)_{t+1} \| , \tag{2.69}$$

where $\| x_{t+1} \| \equiv E[x^2_{t+1}]^{1/2}$. Because y_{t+1} and $M_R(v)_{t+1}$ have the same expectation, the distance between y_{t+1} and $M_R(v)_{t+1}$ in (2.69) is equal to the standard deviation of $y_{t+1} - M_R(v)_{t+1}$, i.e., $\| y_{t+1} - M_R(v)_{t+1} \| = \sigma(y_{t+1} - M_R(v)_{t+1})$. We will denote the stochastic discount factor that solves (2.69) by $\widetilde{m}_R(v)_{t+1}$. Thus, $\widetilde{m}_R(v)_{t+1}$ is the stochastic discount factor that prices R_{t+1} correctly and that is closest to y_{t+1} in a least squares sense.

To solve the problem in (2.69), consider the least square projections of y_{t+1} and $M_R(v)_{t+1}$ on R_{t+1} and a constant:

$$\widehat{y}_{t+1} = \text{Proj}(y_{t+1} \mid 1, R_{t+1}) = v + \zeta(v)'(R_{t+1} - \mu_R), \tag{2.70}$$

$$y_{t+1} = \widehat{y}_{t+1} + u_{t+1},$$

and

$$\begin{aligned} m_R(v)_{t+1} &= \text{Proj}(M_R(v)_{t+1} \mid 1, R_{t+1}) = v + \alpha(v)'(R_{t+1} - \mu_R), \qquad (2.71) \\ M_R(v)_{t+1} &= m_R(v)_{t+1} + w_{t+1}, \end{aligned}$$

where $m_R(v)_{t+1}$ is the minimum variance stochastic discount factor derived in Section 2.2.1, and $\alpha(v)$ is defined in (2.5).The projection coefficients in (2.70) are given by $\Sigma_{RR}^{-1}\Sigma_{Ry}$, with Σ_{Ry} the $K \times 1$-vector of covariances between R_{t+1} and y_{t+1}. Noting that $\| \ y_{t+1} - M_R(v)_{t+1} \ \|^2 = Var[y_{t+1} - M_R(v)_{t+1}]$, it easily follows that

$$\begin{aligned} Var[y_{t+1} - M_R(v)_{t+1}] &= Var[\widehat{y}_{t+1} - m_R(v)_{t+1}] + Var[u_{t+1} - w_{t+1}] \\ &\geq Var[\widehat{y}_{t+1} - m_R(v)_{t+1}]. \end{aligned}$$

Because $\widehat{y}_{t+1} - m_R(v)_{t+1} = y_{t+1} - (m_R(v)_{t+1} + u_{t+1})$ and u_{t+1} is orthogonal to R_{t+1}, this lower bound on the variance of $y_{t+1} - M_R(v)_{t+1}$ is attainable for the stochastic discount factor

$$\widetilde{m}_R(v)_{t+1} = m_R(v)_{t+1} + u_{t+1}, \qquad (2.72)$$

and we have that

$$\delta^2 = Var[y_{t+1} - \widetilde{m}_R(v)_{t+1}]. \qquad (2.73)$$

A more detailed characterization of $\widetilde{m}_R(v)_{t+1}$ and δ will be given in the following section. For this moment, note that subtracting the variable $y_{t+1} - \widetilde{m}_R(v)_{t+1}$ from the proxy y_{t+1} yields a valid stochastic discount factor. Therefore, as noted by Hansen & Jagannathan (1997), $y_{t+1} - \widetilde{m}_R(v)_{t+1}$ is the smallest adjustment in a least squares sense that is necessary to make y_{t+1} a valid stochastic discount factor, and δ is a measure of the magnitude of this adjustment.

Hansen & Jagannathan also show that δ can be interpreted as a maximum pricing error. In order to do so, let ω denote a position in R_{t+1} that does *not* necessarily satisfy the requirement $\omega'\iota_K = 1$, i.e., ω is in general not a portfolio. Denote the payoff of such a position as $R(\omega)_{t+1} = \omega' R_{t+1}$ and note that the correct price of such a positions is

$$E[\omega' R_{t+1} M_R(v)] = \pi(R(\omega)_{t+1}) = \omega'\iota_K,$$

whereas the price assigned by the proxy y_{t+1} is $\pi^a(R(\omega)_{t+1})$. The pricing error of the proxy y_{t+1} is therefore $\pi^a(R(\omega)_{t+1}) - \pi(R(\omega)_{t+1})$, and Hansen & Jagannathan show that δ provides an upper bound on the absolute value of this pricing error, for positions that have a unit norm:

$$\delta = \max_{R(\omega)_{t+1}, \|R(\omega)_{t+1}\| = 1} \mid \pi^a(R(\omega)_{t+1}) - \pi(R(\omega)_{t+1}) \mid .$$

Thus, by looking at a particular class of positions, i.e., positions with a unit norm, δ can be interpreted as the maximum pricing error assigned by the proxy to the payoffs of those unit norm positions.

A more intuitive interpretation can be given if we consider errors in expected payoffs, or expected returns, rather than pricing errors. Recall that a valid stochastic discount factor assigns the correct expected return to a one-dollar investment in portfolio w^p (for which, by definition, $w^{p\prime}\iota = 1$) which, using equation (2.3), can be written as

$$E[R^p_{t+1}] = \frac{1}{v} - \frac{Cov[M_R(v)_{t+1}, R^p_{t+1}]}{v},$$

i.e., as one over the expectation of the pricing kernel, which equals the risk free rate if it exists, plus a risk term which is determined by the covariance of the portfolio return and the pricing kernel. Observe that use of the proxy, that also has expectation v, would give an approximate expected return $E^a[R^p_{t+1}]$ for a one-dollar investment in w^p that in general differs from $E[R^p_{t+1}]$, because the covariance of the proxy with the portfolio return will be different from the covariance of a valid stochastic discount factor with the portfolio return, i.e.:

$$E^a[R^p_{t+1}] = \frac{1}{v} - \frac{Cov[y_{t+1}, R^p_{t+1}]}{v}.$$

From these relations we define the *expected return error*

$$E^a[R^p_{t+1}] - E[R^p_{t+1}] = \frac{Cov[M_R(v)_{t+1} - y_{t+1}, R^p_{t+1}]}{v}, \tag{2.74}$$

for which the Cauchy-Schwarz inequality implies that

$$\mid E^a[R^p_{t+1}] - E[R^p_{t+1}] \mid \leq \frac{\sigma(y_{t+1} - M_R(v)_{t+1})\sigma(R^p_{t+1})}{v}.$$

Since this inequality holds for all valid stochastic discount factors $M_R(v)_{t+1}$, it also holds for the stochastic discount factor that solves (2.69), $\widetilde{m}_R(v)_{t+1}$, implying

$$\mid E^a[R^p_{t+1}] - E[R^p_{t+1}] \mid \leq \frac{\delta\sigma(R^p_{t+1})}{v}.$$

Since for a given value of v, the Sharpe ratio is defined as $Sh(R^p_{t+1}) \equiv (E[R^p_{t+1}] - 1/v)/\sigma(R^p_{t+1})$, and the approximate Sharpe ratio, i.e., the Sharpe ratio according to the proxy y_{t+1}, as $Sh^a(R^p_{t+1}) \equiv (E^a[R^p_{t+1}] - 1/v)/\sigma(R^p_{t+1})$, this can be rewritten as

$$\mid Sh^a(R^p_{t+1}) - Sh(R^p_{t+1}) \mid \leq \frac{\delta}{v}. \tag{2.75}$$

Thus, using errors in expected returns rather than errors in assigned prices, the specification error bound δ scaled by the expectation of the proxy has a very clear interpretation in terms of Sharpe ratios. For any portfolio w^p formed from the assets in R_{t+1}, the absolute difference between the approximate Sharpe ratio assigned to the portfolio returns by y_{t+1} and the actual Sharpe ratio of the portfolio can never exceed the scaled specification error bound δ/v. This interpretation is also somewhat easier than the one given for the expected payoff error in Hansen & Jagannathan (1997), where they focus on the maximum error in expected payoffs for positions ω with unit standard deviation.

2.6.2 The relation between specification error bounds and intersection

The purpose of this section is to show that there is a close relation between intersection and a special case of the specification error bounds. In particular, if the interest is in stochastic discount factors that price the returns (R_{t+1}, r_{t+1}) correctly and choose for the proxy y_{t+1} the minimum variance stochastic discount factor based on the subset R_{t+1}, $m_R(v)_{t+1}$, the specification error bound can simply be expressed as a deviation from intersection, as was the case with the performance measures discussed in Section 2.5. To show this, let us first give a more precise characterization of $\widetilde{m}(v)_{t+1}$ and δ than given in (2.72) and (2.73).

Recall that $\widetilde{m}_R(v)_{t+1}$ is given by $m_R(v)_{t+1} + u_{t+1}$, where $u_{t+1} = y_{t+1} - \widehat{y}_{t+1}$. Using (2.70) and (2.71), this implies for $\widetilde{m}_R(v)_{t+1}$:

$$\begin{aligned}\widetilde{m}_R(v)_{t+1} &= v + \alpha(v)'(R_{t+1} - \mu_R) + y_{t+1} - \{v + \zeta(v)'(R_{t+1} - \mu_R)\}\\ &= y_{t+1} + (\alpha(v) - \zeta(v))'(R_{t+1} - \mu_R)\\ &= y_{t+1} + \{(\iota_K - v\mu_R) - \Sigma_{Ry}\}'\Sigma_{RR}^{-1}(R_{t+1} - \mu_R), \qquad (2.76)\end{aligned}$$

and for δ^2:

$$\delta^2 = \{(\iota_K - v\mu_R) - \Sigma_{Ry}\}'\Sigma_{RR}^{-1}\{(\iota_K - v\mu_R) - \Sigma_{Ry}\}. \qquad (2.77)$$

For further reference it is useful to define the vector $\lambda(v)$ as

$$\lambda(v) = \alpha(v) - \zeta(v) = \Sigma_{RR}^{-1}\{(\iota_K - v\mu_R) - \Sigma_{Ry}\}. \qquad (2.78)$$

Notice that the expressions for $\lambda(v)$ and δ^2 given here differ slightly from the ones given in Hansen & Jagannathan (1997) because we explicitly included a constant in the projections of $M(v)_{t+1}$ and y_{t+1} on R_{t+1}.

The expressions for $\widetilde{m}_R(v)_{t+1}$ and δ^2 in (2.76) and (2.77) provide a basis to relate the specification error bounds to intersection as well as to derive a limit distribution for the sample equivalent of δ. In case of intersection the interest is in stochastic discount factors that price both R_{t+1} and r_{t+1}, i.e., in $M(v)_{t+1}$. Therefore, in the expressions (2.76) and (2.77) we should leave out all the R-subscripts, replace R_{t+1} with the vector $(R'_{t+1}\ r'_{t+1})'$, and note that all vectors and matrices have dimension $K+N$ rather than K. As before, with intersection we want to know if the minimum variance stochastic discount factor based on R_{t+1} only, $m_R(v)_{t+1}$ can be used to price both R_{t+1} and r_{t+1}. In terms of specification errors this means that we want to use $m_R(v)_{t+1}$ as a proxy y_{t+1} for the stochastic discount factors $M(v)_{t+1}$. Also, in the spirit of the previous section, when using $m_R(v)_{t+1}$ as a proxy, we recognize beforehand that $m_R(v)_{t+1}$ will not assign the correct prices to r_{t+1}, but the interest is in the extent to which the assigned prices are wrong, i.e., the extent to which there are deviations from intersection, as measured by δ.

Recall that the proxy $y_{t+1} = m_R(v)_{t+1}$ is now given by

$$\begin{aligned} y_{t+1} &= m_R(v)_{t+1} = v + \alpha_R(v)'(R_{t+1} - \mu_R), \\ \alpha_R(v) &= \Sigma_{RR}^{-1}(\iota_K - v\mu_R). \end{aligned}$$

Substituting these expressions into (2.76) and (2.77), properly adjusted for the fact that the interest is now in stochastic discount factors that price both R_{t+1} and r_{t+1}, straightforward algebra shows that

$$\begin{aligned} \delta^2 &= \{(\iota_N - v\mu_r) - \Sigma_{rR}\Sigma_{RR}^{-1}(\iota_K - v\mu_R)\}'\Sigma_{\varepsilon\varepsilon}^{-1}\{(\iota_N - v\mu_r) - \Sigma_{rR}\Sigma_{RR}^{-1}(\iota_K - v\mu_R)\} \\ &= v^2\alpha_J(1/v)'\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(1/v), \end{aligned} \tag{2.79}$$

or

$$\frac{\delta}{v} = \{\theta(1/v)^2 - \theta_R(1/v)^2\}^{1/2},$$

where $\Sigma_{\varepsilon\varepsilon}$ is the covariance matrix of the residuals ε_{t+1} from a regression of r_{t+1} on R_{t+1} and a constant. Also, the stochastic discount factor closest to y_{t+1} is now given by

$$\widetilde{m}(v)_{t+1} = m_R(v)_{t+1} + v\alpha_J(1/v)'\Sigma_{\varepsilon\varepsilon}^{-1}\varepsilon_{t+1} = m(v)_{t+1}. \tag{2.80}$$

Thus, if we want to use the stochastic discount factor that is on the volatility bound of R_{t+1}, as a proxy stochastic discount factor for the larger set (R_{t+1}, r_{t+1}), then the valid discount factor that is closest to $m_R(v)_{t+1}$ is the discount factor with the same expectation v that is on the volatility bound of (R_{t+1}, r_{t+1}). Therefore, δ is the least squares distance between two stochastic discount factors that are on the volatility bounds of (R_{t+1}, r_{t+1}) and its subset R_{t+1} respectively, and is a straightforward measure of the deviation from intersection, which shows the close relation between this special case of the specification error bound and intersection. This relationship also follows from (2.79), which shows that δ is directly related the change in the maximum squared Sharpe ratios that can be attained with R_{t+1} and (R_{t+1}, r_{t+1}) respectively.

An estimate of δ^2 can easily be obtained from the sample equivalent of (2.77), which we will denote by $\widehat{\delta}^2$. If the interest is in whether or not there is intersection, then we want to know whether or not $\delta = 0$, and this hypothesis can easily be tested as outlined in Section 2.3. From the expression in (2.79) and the discussion in previous sections, it follows that under the null hypothesis that $\delta = 0$,

$$T \frac{\widehat{\delta}^2}{v^2(1+\widehat{\theta}_R(1/v)^2)} \sim \chi^2_N. \tag{2.81}$$

In case of specification errors however, the interest is in the case where δ is strictly positive rather than zero. For that case the limiting distribution of $\widehat{\delta}$ is derived in Hansen, Heaton, & Luttmer (1995). Their derivation is also valid in case there are market frictions such as short sales constraints and transaction costs, but a simplified derivation can be given in case there are no market frictions. Here we will focus on the frictionless case, and postpone the discussion of market frictions until Chapter 4.

To derive the limiting distribution, Hansen & Jagannathan (1997) show that the problem in (2.69) has a dual problem

$$\delta^2 = \max_{\{\lambda(v)\}} \{E[y^2_{t+1}] - E[(y_{t+1} - \lambda(v)'(R_{t+1} - \mu_R))^2] - 2\lambda(v)'(\iota_K - v\mu_R)\}, \tag{2.82}$$

where we restrict ourselves again to R_{t+1} to simplify the notation. If the assets under consideration are (R_{t+1}, r_{t+1}), Equation (2.82) should be adjusted in an obvious way. Notice once more that the dual given here differs slightly from the one given in Hansen & Jagannathan (1997) because of the inclusion of a constant in the projection of $M_R(v)_{t+1}$ onto R_{t+1}. It is easily verified that the first order conditions of (2.82) yield the same solution for $\lambda(v)$ as in (2.78), and that substitution of (2.78) in (2.82) yields (2.77). As in Hansen, Heaton, & Luttmer (1995), define

$$\phi(\lambda)_t \equiv y^2_t - (y_t - \lambda'(R_{t+1} - \mu_R))^2 - 2\lambda'(\iota_K - v\mu_R). \tag{2.83}$$

By definition, the maximizer of $E[\phi(\lambda)_t]$ is λ_0 and the maximizer of $T^{-1}\Sigma_t\phi(\lambda)_t$ is $\widehat{\lambda}$, so

$$\delta^2 = E[\phi(\lambda_0)_t], \text{ and } \widehat{\delta}^2 = T^{-1}\Sigma_t\phi(\widehat{\lambda})_t.$$

Also following Hansen, Heaton, & Luttmer (1995), assume that

$$\frac{1}{\sqrt{T}} \begin{pmatrix} \Sigma_t\{\phi(\lambda_0)_t - E[\phi(\lambda_0)_t]\} \\ \Sigma_t\{(\widetilde{m}(v)_t R_t - \iota_k) - E[\widetilde{m}(v)_t R_t - \iota_k]\} \end{pmatrix}$$

converges in distribution to a normally distributed random vector with mean zero and covariance matrix V, the $(1,1)$-element of which is denoted by $\sigma(\phi)^2$.

In Appendix 2.C it is shown that from this assumption the limiting distribution of $\widehat{\delta}$ can be derived as:

$$\sqrt{T}(\widehat{\delta} - \delta) \xrightarrow{d} N(0, \frac{\sigma(\phi)^2}{4\delta^2}). \tag{2.84}$$

Because the term 4δ in the denominator of the variance of this limit distribution (which is caused by the Taylor series expansion in going from δ^2 to δ), this distribution breaks down if δ is zero. Notice that the hypothesis of intersection implies that $\delta = 0$. Therefore, the asymptotic distribution in (2.84) is not valid under the null hypothesis of intersection. However, the hypothesis that $\delta = 0$ can be tested using (2.81), i.e., with a standard test for intersection.

The interest in Hansen & Jagannathan (1997) is in the case where there is no intersection. Once we concede that $y_{t+1} = m_R(v)_{t+1}$ is not a valid stochastic discount factor for (R_{t+1}, r_{t+1}), we want to have a measure of the difference between $m_R(v)_{t+1}$ and the valid stochastic discount factor that is closest to it, $m(v)_{t+1}$. The specification error bound δ is one such measure with a limit distribution given in (2.84), allowing us to make statements about how good or how bad the proxy performs.

The fact that δ^2 is equal to the change in maximum Sharpe ratios, makes the measure δ also useful in terms of the optimal portfolio choice for a mean-variance investor. Recall that a mean-variance investor that initially only invests in R_{t+1} can improve his Sharpe ratio from $\theta_R(1/v)$ to $\theta(1/v)$ by including r_{t+1} in his portfolio. Given that there is no intersection between the mean-variance frontiers of R_{t+1} and (R_{t+1}, r_{t+1}), $\widehat{\delta}$ provides an estimate for the potential increase in Sharpe ratios. Although such an estimate can also be derived directly from the Wald test-statistic for intersection, the advantage of using $\widehat{\delta}$ is that (2.84) also allows us to make statements about the precision of this estimate. In the next section we will give a short illustration of this by continuing the empirical example of this chapter.

2.6.3 Specification error bounds in the empirical example

To illustrate the specification error bounds discussed above, let us continue the example of Sections 2.3.5 and 2.5.4. The interest is in whether the minimum variance kernel that prices the benchmark assets - the MSCI Indices for the US and Canada - correctly, can also be used to price the Dutch and Japanese indices. As before, let the expectation of the kernel, v, be given by $v = 1/1.0025 = 0.9975$. In that case the minimum variance stochastic discount factor $m_R(v)_{t+1}$ is equal to

$$m_R(0.9975)_{t+1} = 0.9975 - 8.66(R_{t+1}^{US} - 1.0138) + 2.74(R_{t+1}^{Can} - 1.0086).$$

As an aside, notice that the coefficients for the US and Canada are indeed proportional to the portfolio weights in Table 2.3: $-8.66/(-8.66 + 2.74) = 1.46$ and $2.74/(-8.66 + 2.74) = -0.46$, which illustrates the earlier mentioned duality between mean-variance frontiers and volatility bounds.

In Section 2.3.5 it was already shown that the mean-variance frontier of the US and Canada does not intersect the frontier of the US, Canada and the Netherlands at $\eta = 1.0025$. Therefore, the kernel $m_R(0.9975)_{t+1}$ does not price the Dutch index correctly. Although we could not reject the hypothesis of intersection for Japan, in this section we do not impose this hypothesis, and assume that $m_R(0.9975)_{t+1}$ doesn't price the Japanese index correctly either. The specification error bound δ can now be used to make statements about how well $m_R(0.9975)_{t+1}$ prices the Dutch and Japanese indices. Given that we use $m_R(0.9975)_{t+1}$ as a proxy stochastic discount factor y_{t+1} to price all four indices, Table 2.4 presents estimates of the specification error bound δ.

Table 2.4: Estimated specification error bounds

The table reports specification error bounds when using the minimum variance kernel for the US and Canada as a proxy stochastic discount factor for the Netherlands and Japan.

	$\widehat{\delta}$	$std.err.(\widehat{\delta})$	$\widehat{\delta}/v$	$std.err.(\widehat{\delta}/v)$
NL	0.274	(0.090)	0.275	(0.091)
Jap	0.116	(0.088)	0.116	(0.088)
NL + Jap	0.275	(0.091)	0.276	(0.091)

As outlined in the previous section, these bounds can be interpreted in terms of errors in Sharpe ratios. For instance, the estimate of $\widehat{\delta} = 0.274$ for the Netherlands, implies that the difference between the squared Sharpe ratios of the benchmark assets and of the benchmark assets plus the Netherlands is equal to $0.274^2/v^2$. Thus, given that the maximum Sharpe ratio of the benchmark assets is equal to 0.285, this means that the maximum Sharpe ratio when including the Netherlands increases to $\sqrt{0.285^2 + 0.274^2/0.9975^2} = 0.396$.The standard error of the estimated bound, 0.090, gives an idea of the precision of the increase in the Sharpe ratio. Given the initial Sharpe ratio of 0.285 and given the expectation of the kernel of 0.9975, a Taylor series expansion can be used to obtain a standard error for the maximum Sharpe ratio that can be obtained when including the Dutch index in the investment set. This standard error is equal to 0.063, implying that the 95%-confidence interval of the maximum Sharpe ratio is $[0.270, 0.522]$.

Finally, notice from Table 2.4 that the estimated bound for Japan is much smaller than for the Netherlands, although the asymptotic standard errors are approximately the same and that the bound for the Netherlands and Japan together is almost identical to the bound for the Netherlands only.

2.7 Applications

In this section we will discuss some applications of the theoretical framework outlined in the previous sections to some problems that have recently received a lot of attention in the finance literature. These problems concern the diversification benefits of international investments and the efficiency of currency hedging, the diversification benefits of emerging markets, and the three-factor model that has recently been proposed by Fama & French (1996) to explain some well-known CAPM-anomalies. Because these problems are merely meant as an illustration we will not give a complete treatment of them, but only show how they relate to the concepts introduced in this chapter. The applications that we discuss also show that a thorough understanding of the relations between test-statistics for intersection and spanning, performance measures, efficient portfolio weights, and the coefficients in the spanning regression (2.20), can yield useful reinterpretations of many results that have been reported in the literature.

2.7.1 International diversification

It has often been argued that because correlations between stock returns are much lower between countries than within countries, there may be diversification benefits from investing in international stocks as opposed to domestic stocks only (see, e.g., Solnik (1991)). Some evidence for this has already been presented in the empirical examples in Sections 2.3.5, 2.5.4 and 2.6.3. For instance, using the summary statistics in Table 2.1, the Sharpe ratio for the US when the risk free rate is 0.25% per month, is equal to 0.27. When US-investors also invest in Canada, the Netherlands and Japan, the maximum attainable Sharpe ratio increases to 0.40, which illustrates that US-investors can realize diversification benefits from investing in these three countries that are at least economically significant. The increase from 0.27 to 0.40 does not show whether the benefits are also statistically significant however, since these Sharpe ratios suffer from estimation error. To see whether the increase is also statistically significant, we can use the intersection test as presented in Equation (2.66). Given that the results in the example are based on 138 observations, the Wald test-statistic for intersection is:

$$W^{int} = 138 \times \frac{0.40^2 - 0.27^2}{1 + 0.27^2} = 11.20.$$

The p-value associated with this statistic is 0.001, which shows that the increase in the Sharpe ratio is also statistically significant.

DeSantis (1995) uses a similar kind of analysis on a more comprehensive dataset, that also consists of monthly observations of the MSCI Indices over the period July 1973 until December 1992. DeSantis investigates whether it is useful for a US-investor to invest in Europe (Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Norway, Spain, Sweden, Switzerland and the United Kingdom), in Pacific Basin countries (Australia, Hong Kong, Japan and Singapore) and in Canada. The risk free rate is taken to be equal to 0.62% per month, which is derived from the expectation of the kernel that is on the minimum of the volatility bound for the US only. The empirical results of DeSantis (1995) are for the null-hypothesis that the mean-variance frontier (volatility bound) of the US intersects the mean-variance frontier (volatility bound) of the US plus the set of European countries, the Pacific Basin countries, or a Global set (Europe + Pacific Basin + Canada). DeSantis reports the increase in the volatility bound, i.e., in $\sigma(m(v)_{t+1})$. However, from Equations (2.7) and (2.55) it can easily be seen that

$$Var[m(v_{t+1})] = v^2\theta(1/v)^2.$$

Thus, because of the duality between volatility bounds and mean-variance frontiers it is straightforward to obtain the increase in the Sharpe ratios from the results in DeSantis (1995). The tests used by DeSantis whether the shifts in the volatility bounds (mean-variance frontiers) are statistically significant are based on the GMM-test for overidentifying restrictions described at the end of Section 2.3.4. Because the reported Sharpe ratio for the US only is 0.089, it is also possible to derive the Wald test-statistic for intersection as in (2.66) from these results. The results reported by DeSantis, as well as the derived results in terms of Sharpe ratios and the Wald test for intersection, are reported in Table 2.5. These results are based on unhedged US-dollar based returns.

Table 2.5: Intersection tests for international diversifaction

The table presents intersection tests for international stock portfolios as reported by DeSantis (1995) as well as the implied changes in Sharpe ratios. The MSCI Index for the US is the benchmark. Europe = Austria, Belgium, Denmark, France, Germany, Italy, the Netherlands, Norway, Spain, Sweden, Switzerland and the United Kingdom; Pacific Basin = Australia, Hong Kong, Japan and Singapore; Global = Europe + Pacific Basin + Canada. Results are based on monthly unhedged US-dollar based returns for the period from July 1973 until December 1992. The intersection tests are based on $v = 1.0062$. Δbound is the change in the volatility of the minimum variance kernel with expectation v; ΔSharpe is the change in the (maximum) Sharpe ratio for $\eta = 1/v$.

	Δbound	ΔSharpe	p-value (GMM)	Wald	p-value (Wald)
Europe	0.103	0.103	(0.847)	6.74	(0.875)
Pacific Basin	0.048	0.048	(0.744)	2.51	(0.643)
Global	0.135	0.135	(0.934)	9.85	(0.910)

First of all, notice that because v is close to one, the reported change in the volatility bound (Δbound) is close to the increase in the Sharpe ratio (ΔSharpe). From an economic point of view, the reported changes in the Sharpe ratios are quite large, suggesting that there are large diversification benefits possible from international investments. However, the p-values associated with both the GMM-based test and the Wald test show that the increase in the Sharpe ratios are not statistically significant. Notice that in case of Europe for instance, the increase in the Sharpe ratio by 0.103 is obtained after adding 12 countries. Therefore, both the Wald test-statistic and the GMM test-statistic are asymptotically χ^2_{12}-distributed under the null-hypothesis of intersection. With a sample size of 234 observations, the observed increase in Sharpe ratios is not sufficient to reject the hypothesis of spanning with this number of degrees of freedom.

Similar results can also be derived from Glen & Jorion (1993), who use monthly unhedged US-dollar based returns on the MSCI Indices in excess of the US T-Bill rate for the period January 1974 until December 1990. Although the main interest in Glen & Jorion (1993) is on the benefits of hedging the currency exposure of international stock and bond portfolios, from their results we can also derive some conclusions about the benefits of international diversification. For instance, from their summary statistics it follows that the (monthly) Sharpe ratio of US stocks is 0.079 for their sample period. When adding the MSCI Indices for Germany, Japan, the United Kingdom and France, the maximum attainable Sharpe ratio increases to 0.166. Since there are 204 observations, this implies for the Wald test-statistic for intersection

$$W^{int} = 204 \times \frac{0.166^2 - 0.079^2}{1 + 0.079^2} = 4.33.$$

The p-value associated with this test is 0.363, implying that the increase in Sharpe ratios from 0.079 to 0.166 is again not statistically significant. This is also the case when both stocks and bonds from all countries are considered together. A mean-variance efficient portfolio of stocks and bonds from these five countries yields a Sharpe ratio of 0.184. If the null-hypothesis is that all these 10 securities are spanned by the MSCI Index for US stocks (plus T-Bills) only, then the Wald test-statistic is equal 5.61. Given that there are 9 securities added to the portfolio, the Wald test-statistic is asymptotically χ^2_9-distributed. The p-value for the test-statistic of 5.61 is therefore 0.778, showing that even when both domestic and international bond portfolios are added to the US-index, there is no (statistically) significant increase in the Sharpe ratio.

All these results are based on unhedged US-dollar based returns. As noted, in Glen & Jorion (1993) the interest is in the benefits of hedging currency risk associated with foreign investments. They show that there are significant diversification benefits, both statistically and economically, from including forward contracts in a portfolio of international bonds or stocks and bonds, but not in a portfolio of international stocks only. For instance, for a US-investor that initially invests in the stocks and bonds of the five countries mentioned earlier, including forward contracts on the four currencies (German Mark, Japanese Yen, British Pound, and French Franc) causes an increase in the Sharpe ratio from 0.184 to 0.299. Thus, the null-hypothesis is that the mean-variance frontier of the 10 stock and bond indices intersects (spans) the mean-variance frontier of these same indices plus four currency forward contracts, when the risk free rate is equal to the one month US T-Bill rate. From the reported Sharpe ratios, the Wald test-statistic for this null-hypothesis 10.96 with a p-value of 0.027. In Chapter 3 we will show how forward contracts can be included directly in the regression framework for testing for mean-variance spanning and intersection and how we can test whether or not hedging is beneficial for fixed portfolios rather than the portfolios considered by Glen & Jorion, where the optimal bond, stock and forward positions are chosen simultaneously.

Notice though, that Glen & Jorion (1993) assume that the investor will in any case choose his portfolio from the stocks and bonds of all these five countries. We already saw above that we can not reject the null hypothesis that the mean-variance frontier of the MSCI Index for US stocks intersects (spans) the mean-variance frontier of the stocks and bonds of all five countries. This suggests that the diversification benefits of the forward contracts may be much larger than the diversification benefits of the international stocks and bonds. It is therefore natural to ask if the US-investor can benefit from adding international stocks and bonds as well as the currency forwards to his portfolio of US stocks only. In other words, can we reject the null-hypothesis that the frontier of the MSCI Index for US stocks intersects (spans) the frontier of the stocks and bonds of all five countries *plus* the four forward contracts? This would induce an increase in the Sharpe ratio from 0.079 to 0.299, which is significant in economic terms. However, notice that this increase is obtained by adding 13 securities to the MSCI Index for the US. The Wald test-statistic for intersection of 16.87 is therefore asymptotically χ^2_{13}-distributed, implying a p-value of 0.205. Therefore, this latter hypothesis can not be rejected.

Summarizing, although the example in the previous sections suggests otherwise, based on the evidence reported in DeSantis (1995) and Glen & Jorion (1993) we can not reject the hypothesis that the mean-variance frontier of the MSCI Index of the US intersects the mean-variance frontier of this same index plus a number international stock and bond portfolios and forward contracts. The difference between the results reported in DeSantis (1995) and Glen & Jorion (1993) on the one hand and the results of the example used in the previous sections on the other, is caused by the fact that the empirical example in this chapter uses data until 1996. During the last few years of our sample period, stock returns have been especially very high in the Netherlands, causing the large diversification benefits shown in the example.

In economic terms, the increase in the Sharpe ratio that may be obtained from international diversification as reported by DeSantis (1995) and Glen & Jorion (1993) is often impressive. Given the number of observations in these studies and the number of securities that is added to the portfolio, these increases are not statistically significant however. On the other hand, for an investor who has invested in a portfolio of unhedged international stocks and bonds, adding forward contracts to hedge his currency exposure yields an increase in the Sharpe ratio that is both economically and statistically significant. Also, although not reported here, DeSantis (1995) shows that the hypothesis of intersection can be rejected when including managed portfolios, i.e., when incorporating conditional information (the lagged return on the World portfolio, the lagged dividend yield on the World portfolio and the term premium in the US bond market). Similarly, Glen & Jorion (1993) show that the benefits from currency hedging are much more profound when the hedging strategy is conditional on the forward premium (i.e., the interest differential between two countries).

2.7.2 Emerging markets

The results in the previous section showed that the benefits of international diversification to a US-investor, although often impressive in economic terms, are usually not statistically significant. However, in the studies of DeSantis (1995) and Glen & Jorion (1993) the focus is on portfolios consisting of investments in the US as well as a number of well-developed equity markets, such as the German, Japanese and UK markets. The past twenty years have witnessed the emergence of many new equity markets in Europe, Latin America, Asia, the Mideast and Africa that offer new investment opportunities to investors. These emerging markets have been characterized by both high average returns and high volatility, but low correlations with equity returns in the developed markets. Therefore, although these emerging markets in themselves appear to provide risky investments, they may also provide substantial diversification benefits to US-investors.

For instance, Harvey (1995) reports an annualized average return of 20.36% for the Composite Index for emerging markets of the International Finance Corporation (IFC) over the period February 1985 until June 1992. The annualized standard deviation of this index is 24.70%. For the period February 1976 until June 1992 the average return on the MSCI World Index was 13.91%, and the standard deviation 14.36%. Moreover, the annualized average return for the individual emerging markets over the period February 1976 until June 1992 is in the range between 9.43% (Greece) and 71.79% (Argentina). The standard deviations during this period are in the range between 25.67% (Thailand) and 105.06% (Argentina). These statistics are all based on US dollar-based returns and show that the emerging markets are indeed characterized by high average returns and high volatility. As with respect to correlations, Harvey (1995) reports that the average cross country correlation in 18 developed markets is 0.41, whereas the average cross country correlation in 20 emerging markets is only 0.12. Furthermore, the average correlation between emerging markets and developed markets is 0.14, suggesting that there may be large diversification benefits from investing in emerging markets. This is confirmed by a comparison of the mean-variance frontiers of 18 developed countries and of 18 developed countries and 18 emerging markets, as presented in Harvey (1995). For the developed countries only, the global minimum-variance portfolio has a standard deviation of 13%. Adding the 18 emerging markets results in a global minimum-variance portfolio with a standard deviation of only 7%

Recall from Equation (2.67) in Section 2.5.3 that the Wald test-statistic for spanning can be decomposed into a part that is determined by the change in the global minimum variance and a part that is determined by the slope of the asymptotes along the frontier. These terms are always nonnegative and additive, so a lower bound for the Wald test-statistic for spanning can be derived by calculating the part in (2.67) that depends on the global minimum variances. Given that the frontiers are calculated using a time series of 75 observations, the lower bound on the test-statistic is

$$\xi_W^{span} \geq T\left(\frac{(\widehat{\sigma}_R^0)^2}{(\widehat{\sigma}^0)^2} - 1\right) = 75 \times \left(\frac{0.13^2}{0.07^2} - 1\right) = 183.7.$$

The p-value associated with a χ^2 with 36 degrees of freedom (18 emerging markets) is 0.000, implying that the hypothesis of spanning can easily be rejected. Harvey (1995) similarly rejects the null hypothesis that there is intersection at some point for the two frontiers at any conventional significance level. Therefore, unlike the developed markets, the emerging markets appear to offer diversification benefits to US-investors that are economically and statistically significant.

These results are further corroborated by testing a one-factor model where the cross-section of expected returns on the emerging markets is explained by their covariation with the world portfolio. Specifically, Harvey tests whether the intercepts in the regressions

$$r_{i,t+1} - R_t^f = \alpha_i + \beta_i(R_{t+1}^{world} - R_t^f) + \varepsilon_{i,t+1} \tag{2.85}$$

are equal to zero for all i. Here $r_{i,t+1}$ is the return on emerging market i, R_{t+1}^{world} is the return on the MSCI World Index, and R_t^f is the return on a 30-days Eurodollar deposit. Harvey motivates the use of (2.85) by the World CAPM, which implies that $\alpha_i = 0$ for each emerging market i. From the results in Section 2.5 it is clear that α_i is the Jensen measure for emerging market i relative to the world portfolio. The test whether or not $\alpha_i = 0$ is also a test whether there is intersection with the world portfolio as the benchmark asset and the zero beta rate equal to the risk free rate. Thus, instead of motivating (2.85) by the World CAPM and testing whether stock returns in emerging markets can be explained by their covariation with the world portfolio, (2.85) can also be motivated by the question whether an investor that initially holds the MSCI World Index (plus the risk free asset) can improve his efficient set by also investing in emerging markets.

For the individual emerging markets, the estimated annualized intercepts are in the range between -16.65% (Indonesia) and 63.42% (Argentina) and 5 out of 20 intercepts are significantly different from zero (Argentina, Chile, Colombia, Pakistan and Philippines). Similar results will be reported for a sample period that ends in June 1996 in Chapter 4. The regression estimates as reported by Harvey (1995) can be used to obtain information about the attainable Sharpe ratios and the new optimal portfolio weights for the assets considered. This will be illustrated in detail in the next Section for the Fama & French three-factor model. A joint test whether the intercepts of all 18 emerging markets are zero is rejected at any conventional significance level (p-value is 0.001).

Thus the hypothesis that the MSCI World Index spans the 18 emerging markets is convincingly rejected, implying that there are significant diversification benefits to an investor that initially only holds the world portfolio and that the World CAPM can not explain the cross section of emerging market returns.

Harvey (1995) also tests whether the intercepts in a two-factor model are equal to zero, i.e., whether in the regression

$$r_{i,t+1} - R_t^f = \alpha_i + \beta_{1,i}(R_{t+1}^{world} - R_t^f) + \beta_{2,i}(R_{t+1}^{FX} - R_t^f) + \varepsilon_{i,t+1}, \tag{2.86}$$

$\alpha_i = 0.\forall i$. Here R_{t+1}^{FX} is the return on a trade-weighted portfolio of Eurocurrency deposits in 10 countries. The regression in (2.86) is motivated by the international asset pricing model of Adler & Dumas (1983). The estimated (annualized) intercepts for this model are in the range between -12.97% (Indonesia) and 64.06% (Argentina). Again, 5 out of the 20 intercepts are significantly different from zero (Argentina, Chile, Colombia, Philippines and Taiwan) and the p-value associated with a test for the hypothesis that the intercepts of all 18 emerging markets are zero is 0.001. Thus, the model of Adler & Dumas can not explain the cross section of emerging market returns either and US-investors that initially hold the world portfolio plus a trade-weighted portfolio of Eurocurrency deposits can extend their efficient set significantly by investing in the emerging markets.

Similar conclusions about the diversification benefits of emerging markets are reported by DeSantis (1994) for instance. As noted by Bekaert & Urias (1996) however, a drawback of many studies on the diversification benefits of emerging markets is that the IFC Global Indices that are used in studies on emerging markets ignore the high transaction costs, low liquidity and investment constraints associated with emerging markets. Therefore, the diversification benefits suggested by these studies may not be attainable in real life. Bekaert & Urias try to overcome this problem by using the returns on emerging market closed-end country funds. Since these country funds are traded in the US-market itself for instance, they provide an indirect investment opportunity in emerging markets that is attainable to US-investors. Based on emerging market country funds in the US and the UK, Bekaert & Urias (1996) find only mixed evidence for the diversification benefits of emerging markets. This suggests that market frictions such as transaction costs and short sales constraints in the emerging markets may indeed prevent investors from realizing the diversification benefits of emerging markets. In Chapter 4 we will study the effect of short sales constraints and transaction costs on tests for spanning and intersection in more detail and analyze the consequences of such market frictions for direct investments in emerging markets.

2.7.3 The Fama-French three-factor asset pricing model

In a recent paper Fama & French (1996) propose a three-factor model to explain cross-sectional variations in asset returns. It is well known that the static CAPM can not explain many patterns in stock returns that are related to size, book-to-market equity (BE/ME), cash flow/price (C/P), earnings/price (E/P), and past sales growth. Also, stocks with low returns in the long-term (five year) past appear to have high expected future returns and stocks that have had a high return in last year also have high expected future returns (momentum), findings that can not be explained by the CAPM.

To illustrate these kinds of effects, Fama & French (1996) sort the NYSE, AMEX and Nasdaq stocks based on, e.g., their E/P at the end of June of each year. These stocks are then allocated to ten portfolios, based on the decile breakpoints for E/P. For each of these ten portfolios monthly returns (equal weighted or value weighted) are calculated from July until the next June. This procedure is repeated for each year from July 1963 until December 1993. In a similar way, portfolios are formed based on BE/ME deciles, C/P deciles etc. For some variables also double sort portfolios are constructed. For instance, when sorting on BE/ME and (past) Sales, Fama & French sort the stocks independently on the basis of three BE/ME groups and three Sales groups, resulting in a total of 9 portfolios.

Denote the return on a portfolio as $r_{i,t+1}$. Given a risk free rate R_t^f and the return on the market portfolio, R_{t+1}^m, the CAPM implies that in the regression

$$r_{i,t+1} - R_t^f = \alpha_i + \beta_i(R_{t+1}^m - R_t^f) + \varepsilon_{i,t+1} \tag{2.87}$$

$\alpha_i = 0, \forall i$. Notice that α_i is simply the classical Jensen measure. In other words, according to the CAPM the market portfolio and the risk free asset span all assets or portfolios i, as outlined in Section 2.3. Thus, with a risk free asset available (which in Fama & French (1996) is the one-month T-Bill rate), a test for the validity of the CAPM is simply a test whether the market portfolio intersects (spans) all other assets or portfolios in the economy. For each set of portfolios (i.e., based on a particular sort), Table 2.6 presents the average absolute intercept of the regression in (2.87) as well as the Gibbons-Ross-Shanken (GRS) test for zero-intercepts in (2.87). As noted in Section 2.5.3, the GRS-test is the small-sample version of the test in (2.66).

The 10 E/P sorted portfolios produce an average (absolute) intercept of 0.260. The GRS-test, which is calculated as

$$GRS = \frac{T-N-K}{N} \frac{\widehat{\theta}(R_t^f)^2 - \widehat{\theta}_R(R_t^f)^2}{1+\widehat{\theta}_R(R_t^f)^2}, \tag{2.88}$$

Table 2.6: Summary of intercepts and of spanning tests based on the CAPM.

The results in the table are taken from Table IX in Fama & French (1996). Average absolute intercepts, intersection tests, changes in Sharpe ratios, and specification error bounds are shown when sorted portfolios are added to the market portfolio. Portfolios are sorted on size and book-to-market equity (double sort), earnings-price, past sales growth, cash flow/price and past sales growth (double sort), long-term past returns, which are from 60-13 months before formation, and short-term past returns, which are from 12-2 months before formation.

Portfolio	Avg.($\lvert\alpha_i\rvert$)	GRS	*p*(GRS)	ΔSharpe	$\widehat{\delta/v}$
Size&BE/ME	0.286	2.76	(0.000)	0.362	0.453
E/P	0.260	2.85	(0.002)	0.201	0.285
Sales	0.256	2.51	(0.006)	0.184	0.267
C/P&Sales	0.268	2.93	(0.002)	0.190	0.274
returns(-60,-13)	0.268	2.51	(0.006)	0.184	0.267
returns(-12,-2)	0.337	5.13	(0.000)	0.293	0.382

and which is (under the null-hypothesis of spanning and if all asset returns are jointly normally distributed and $i.i.d.$) $F_{T-N-K,N}$-distributed, is equal to 2.85 for the E/P-based portfolios. The summary statistics in Fama & French (1996) imply that the Sharpe ratio of the market portfolio is $\widehat{\theta}_R(R_t^f) = 0.102$. Since there is only one benchmark asset (the market portfolio), we have $K = 1$, and for the E/P-portfolios we have $N = 10$. Given that the sample size is 366, it follows that investing in the ten E/P-based portfolios besides the market portfolio, causes an increase in the maximum attainable Sharpe ratio from 0.102 to 0.302, which is not only significant in statistical terms, but also in economic terms. Similar results are also reported for the other sorts in Table 2.6.

The results in Table 2.6 illustrate some well-known CAPM-anomalies: empirical regularities in stock returns that can not be explained by the static CAPM. Fama & French (1996) claim that most of these anomalies are captured by their three-factor model, which states that the expected excess return on portfolio i is

$$E[r_{i,t+1} - R_t^f] = \beta_i^m E[R_{t+1}^m - R_t^f] + \beta_i^s E[R_{t+1}^{SMB}] + \beta_i^h E[R_{t+1}^{HML}], \tag{2.89}$$

where R_{t+1}^m and R_t^f are defined as before, R_{t+1}^{SMB} is the difference between a return on a portfolio of small stocks and the return on a portfolio of big stocks, and where R_{t+1}^{HML} is the difference between the return on a portfolio of high book-to-market stocks and the return on a portfolio of low book-to-market stocks. The small, big, high book-to-market and low book-to-market portfolios are created in a similar way as the portfolios described above. According to (2.89) expected stock returns are not only explained by the covariance of stock returns with the market (β_i^m) as the CAPM predicts, but also by their covariation with R_{t+1}^{SMB} and R_{t+1}^{HML}. The loading on $E[R_{t+1}^{SMB}]$, β_i^s, captures the well known size-effect. Small firms have average returns that can not be captured by the market return (see, e.g., Huberman & Kandel (1987)). Fama & French (1996) interpret $\beta_i^h E[R_{t+1}^{HML}]$ as a premium for relative distress of a firm. They claim that weak firms tend to have high BE/ME ratios and positive slope coefficients β_i^h. Because $E[R_{t+1}^{HML}] > 0$, firms in distress will have higher expected returns.

Notice that R_{t+1}^{SMB} and R_{t+1}^{HML} are zero-investment positions. However, if these positions are combined with an investment in the risk free asset, then portfolios are created with return $R_{t+1}^{SMB} + R_t^f$ and $R_{t+1}^{HML} + R_t^f$ respectively. We will refer to these portfolios as the SMB-portfolio and the HML-portfolio. R_{t+1}^{SMB} and R_{t+1}^{HML} can therefore be interpreted as the excess returns on these portfolios. From Section 2.3 it follows that Equation (2.89) implies that the mean-variance frontier of the market portfolio and the SMB and HML-portfolios intersects (spans) the mean-variance frontier of these same portfolios plus all other portfolios, for a known risk free rate R_t^f. Therefore, the model in (2.89) can be tested by testing whether in the regression

$$r_{i,t+1} - R_t^f = \alpha_i + \beta_i^m(R_{t+1}^m - R_t^f) + \beta_i^s R_{t+1}^{SMB} + \beta_i^h R_{t+1}^{HML} + \varepsilon_{i,t+1}, \tag{2.90}$$

$\alpha_i = 0, \forall i$. Notice again that α_i is the generalized Jensen measure for the Fama & French three-factor model. Because Fama & French (1996) also present the GRS-tests based on (2.90), we can construct a table similar to Table 2.6 for the three-factor model as well. From the summary statistics the maximum Sharpe ratio for the market portfolio and the SMB and HML-portfolios can be calculated, which is 0.261. Since there are now three benchmark portfolios, $K = 3$. Using the reported GRS test-statistics in Fama & French (1996) it is then straightforward to calculate the increase in the maximum Sharpe ratios that may result from adding the portfolios based on the sorts in Table 2.6. These calculations are reported in Table 2.7.

Table 2.7: Summary of intercepts and of spanning tests based on the Fama-French three-factor model.

The results in the table are taken from Table IX in Fama & French (1996). Average absolute intercepts, intersection tests, changes in Sharpe ratios, and specification error bounds are shown when sorted portfolios are added to the three factor portfolios of Fama & French (1996) (market, SMB, and HML). Portfolios are sorted on Size and book-to-market equity (double sort), earnings-price, past sales growth, cash flow/price and past sales growth (double sort), long-term past returns, which are from 60-13 months before formation, and short-term past returns, which are from 12-2 months before formation.

Portfolio	Avg.($\lvert\alpha_i\rvert$)	GRS	p(GRS)	ΔSharpe	$\hat{\delta}/v$
Size&BE/ME	0.093	1.97	(0.004)	0.212	0.395
E/P	0.051	0.84	(0.592)	0.045	0.159
Sales	0.053	0.87	(0.563)	0.046	0.162
C/P&Sales	0.062	1.04	(0.405)	0.049	0.168
returns(-60,-13)	0.092	1.29	(0.235)	0.066	0.198
returns(-12,-2)	0.331	4.46	(0.000)	0.190	0.367

The increases in the Sharpe ratios in Table 2.7 are much smaller than in Table 2.6. For instance, adding portfolios based on an E/P-sort to the market portfolio only, yields an increase in the Sharpe ratio of 0.201 in Table 2.6. Starting from the market portfolio and the SMB and HML-portfolios, adding the E/P-based portfolios yields an increase in the Sharpe ratio of only 0.045. Also, as the GRS-test shows, this latter increase is not statistically significant. From the discussion in Section 2.6.2, recall that the specification error bound introduced by Hansen & Jagannathan (1997) is a function of the Sharpe ratios and the expectation of the kernel, v: $\delta = v(\theta(1/v)^2 - \theta_R(1/v)^2)^{\frac{1}{2}}$. The value of $\widehat{\delta}/v$ is reported in the last columns of Table 2.6 and 2.7. Notice that v is not reported by Fama & French (1996), but v will be close to one, so $\widehat{\delta} \approx \widehat{\delta}/v$. Except for the last rows of Table 2.6 and 2.7, which will be discussed in detail below, notice that the specification error bounds are much smaller in Table 2.7 than in Table 2.6. The reported bounds in Table 2.6 are mostly of the same size as the bounds reported for the market model by Hansen & Jagannathan (1997), which are approximately 0.29. Thus, the specification error bounds confirm that the three-factor model shows less misspecification than the CAPM, although the bounds in Table 2.7 are still rather large. For instance, the bound derived from the P/E-based portfolios in table is 0.159, implying that in constructing portfolios from the three benchmark assets and the ten P/E-based portfolios, the three-factor model may imply a Sharpe ratio that is as far off as 0.159. Unfortunately, the results in Fama & French (1996) do not allow us to make an estimate of the standard error associated with this bound.

Although the results in Table 2.6 and 2.7 show that the three-factor model is much better able to explain expected stock returns than the static CAPM, there is still some evidence left against the three-factor model. First, the double-sorted portfolios on Size and BE/ME give an increase in the Sharpe ratio of 0.212 that is both economically and statistically significant, as the first row of Table 2.7 shows. The double sort on Size and BE/ME in Fama & French (1996) results in 25 portfolios. A closer look at the results in Fama & French (1996) shows that the three-factor model can explain most of the variation in portfolio returns, except for the smallest size stocks with the lowest BE/ME ratios, which have a large negative $\widehat{\alpha}_i$, and the largest size stocks with the lowest BE/ME ratios, which have a large positive $\widehat{\alpha}_i$. For the other portfolios the estimated $\widehat{\alpha}_i$ is close to zero. The main failure of the three-factor model is in explaining returns for portfolios based on short-term past returns, as the last row of Table 2.7 shows. The portfolios labelled (-12,-2) are sorted on the return in the period 2-12 months prior to portfolio formation. These portfolios are meant to capture *momentum strategies* or *continuation* of short term returns. As shown in Table 2.7, investing in these portfolios besides investment in the three benchmark portfolios gives a significant improvement of the efficient set. Also, the results in Fama & French (1996) show that this improvement is almost uniform over the ten portfolios that are formed on the basis of returns in the period (-12,-2).

Finally, the results in Fama & French (1996) can be used to infer mean-variance efficient portfolios. The three-factor model suggests that (mean-variance) investors only have to invest in the market and the SMB and HML-portfolios and the risk free asset. Given the summary statistics in Fama & French (1996), the portfolio weights in the tangency portfolio can easily be calculated using standard mean-variance analysis (see, e.g., Ingersoll (1987), p.88-89). These weights are shown in the first column of Table 2.8. The expected excess return on the tangency portfolio is equal to 0.43% per month and the standard deviation of the portfolio return is 1.65%. As noted above, the short-term continuation portfolios are formed each year based on returns in

Table 2.8: Portfolio weights for tangency portfolios of the Fama / French factor portolios and of two short-term continuation protfolios.

	3-factor	+ cont(1)	+ cont(10)
M	0.25	0.35	-0.29
SMB	0.15	0.34	-0.17
HML	0.61	0.38	0.31
cont(1)		-0.21	
cont(10)			0.37

the period 12-2 months before formation. Denote the return on the portfolio of stocks in the decile with the lowest returns as $r_{t+1}^{(1)}$ and the return on the portfolio of stocks in the decile with the highest return as $r_{t+1}^{(10)}$. The estimated regressions for these portfolios as reported in Fama & French (1996) are:

$$\begin{aligned} r_{t+1}^{(1)} - R_t^f &= -1.15 + 1.14(R_{t+1}^m - R_t^f) + 1.35R_{t+1}^{SMB} + 0.54R_{t+1}^{HML} + \varepsilon_{t+1}^{(1)}, \\ r_{t+1}^{(10)} - R_t^f &= 0.59 + 1.13(R_{t+1}^m - R_t^f) + 0.68R_{t+1}^{SMB} + 0.04R_{t+1}^{HML} + \varepsilon_{t+1}^{(10)}. \end{aligned}$$

Recall that the intercepts are the generalized Jensen measures, and that the sign of the intercept determines whether a mean-variance investor can improve the Sharpe ratio of his portfolio by taking a long or a short position. Thus, besides investing in the market and the SMB and the HML-portfolios, the investor can extend his efficient set by going short in the lowest decile portfolio (with return $r_{t+1}^{(1)}$) or by buying the highest decile portfolio (with return $r_{t+1}^{(10)}$).

Keeping the expected excess portfolio return constant at 0.43%, the new optimal portfolio weights can be determined using (2.62) and (2.63). The required estimate of $\sigma(\varepsilon)$ that is needed for these weights can be derived from the Sharpe ratio of the three benchmark portfolios (0.261) and the t-values of the intercept, which for the lowest decile portfolio is -5.34 and for the highest decile portfolio 4.56. Using (2.65) it follows that $\sigma(\varepsilon)$ is 3.99% and 2.32% for the lowest and the highest decile portfolios respectively. Given these estimates, the optimal portfolio weight for the lowest decile portfolio is equal to

$$w_r^{(1)} = \frac{0.43\%}{(0.261^2 + (-1.15\%/3.99\%)^2)} \times \frac{-1.15\%}{3.99\%^2} = -0.21.$$

Thus, to obtain the new maximum Sharpe ratio and have an expected excess portfolio return of 0.43%, the investor will need to take a short position in the lowest decile portfolio of 0.21. The funds obtained from selling this portfolio short are invested in the market portfolio and the SMB-portfolio, while he will also sell part of his holdings in the HML-portfolio, as can be seen in the second column of Table 2.8. Similar results are also reported for the highest decile portfolio in the third column of Table 2.8. Notice that both the benchmark portfolio and the continuation-based portfolios may contain any of the available stocks. Therefore, it is not clear from the reported results whether or not short positions in the individual stocks are required to realize the increase in the Sharpe ratios.

Summarizing, the results in Fama & French (1996) show that investors that initially hold the market portfolio of US stocks, can significantly improve their portfolio performance by using strategies based on well documented CAPM-anomalies, such as E/P, BE/ME, winner/loser and momentum strategies. This is not the case for investors that base their portfolio on the Fama & French three-factor model however. Investors that initially choose their mean-variance efficient portfolio from the market, the SMB and the HML-portfolios can not reject the efficiency of their portfolio with respect to most of the strategies mentioned. The main exception appears to be caused by momentum strategies: Investing in portfolios that are formed on the basis of short-term past performance causes a shift in the mean-variance frontier of the three benchmark portfolios that is significant in both economic and statistical terms.

The applications given in this section are merely a brief illustration of the way in which some recent results in the empirical finance literature can be interpreted. Other applications can be found in the literature on performance evaluation, hedge funds, and (commodity) futures pricing for instance. It is far beyond the scope of this thesis to give a detailed discussion on all of these applications. The purpose here is just to show how ideas related to mean-variance spanning and intersection can be used to interpret empirical tests on asset pricing and portfolio choice using fairly simple techniques.

2.8 Concluding remarks

The purpose of this chapter was to introduce and illustrate the idea of mean-variance spanning and intersection. In the following three chapters these ideas will be extended and applied to emerging markets and to futures markets. In the next chapter it will be shown how futures (forward) contracts and fixed (nonmarketable) positions can be incorporated in regression based tests for spanning. As discussed above, Glen & Jorion (1993) analyzed the question whether or not to include currency forwards in international stock and bond portfolios using Sharpe ratios. In chapter 3 we will show how futures contracts can be incorporated in the regression framework directly and how to account for fixed asset positions. Moreover, in that chapter it will also be shown how we can test for spanning for non mean-variance utility functions. These results will then be applied to a set of commodity and currency futures that are added to a set of three international stock indices that serve as the benchmark assets. In Chapter 4 the ideas in this chapter will be extended to allow for market frictions such as short sales constraints and transaction costs. There it will be shown that the regression framework to test for intersection and spanning can easily be extended to account for short sales constraints and transaction costs. Because these market frictions may especially be a problem in emerging markets as noted by Bekaert & Urias (1996), these tests will subsequently be applied to the emerging markets database used by Harvey (1995) and DeSantis (1994). Part II of this thesis, which focuses on risk premia in futures markets, also builds on the material in this chapter. In modeling risk premia in futures markets, in Chapter 6 we will analyze the misspecification of several futures pricing models using the specification error bound introduced by Hansen & Jagannathan (1997) that was discussed in Section 6 of this chapter.

Appendix 2.A Consistency of the OLS-estimates when incorporating conditioning variables

The purpose of this appendix is to prove the result referred to in Section 2.4.2. Let the asset returns be described by

$$\begin{aligned} R_{t+1} &= \gamma_R' Z_t + \varepsilon_{R,t+1}, \\ r_{t+1} &= \gamma_r' Z_t + \varepsilon_{r,t+1}, \end{aligned}$$

with $E[\varepsilon_{R,t+1}] = E[\varepsilon_{R,t+1} x_t] = 0$, $E[\varepsilon_{r,t+1}] = E[\varepsilon_{r,t+1} x_t] = 0$, and $\varepsilon_{R,t+1}$ and $\varepsilon_{r,t+1}$ jointly *i.i.d.* with variances and covariances given by Ω_{RR}, Ω_{rr}, and Ω_{rR}, then in the regression

$$R_t^f = \gamma x_t + \delta R_{t+1} + u_{t+1},$$

with $E[u_{t+1}] = 0$, $E[u_{t+1} x_t] = 0$, and $E[u_{t+1} R_{t+1}] = 0$, the OLS-estimates $\widehat{\gamma}$ and $\widehat{\delta}$ are given by

$$\widehat{\gamma} = (\widehat{\gamma}_r' - \widehat{\gamma}_R' \widehat{\Omega}_{RR}^{-1} \widehat{\Omega}_{Rr}) \text{ and } \widehat{\delta} = \widehat{\Omega}_{RR}^{-1} \widehat{\Omega}_{Rr}.$$

To see this, first rewrite the regression model as

$$r = \begin{pmatrix} X & R \end{pmatrix} \begin{pmatrix} \gamma \\ \delta \end{pmatrix} + u,$$

with r a $T \times N$ matrix, X a $T \times (L+1)$ matrix, R a $T \times K$ matrix, γ a $(L+1) \times N$ matrix, δ a $K \times N$ matrix, and u a $T \times N$ matrix of error terms. Define the idempotent matrix M as

$$M = I_T - X(X'X)^{-1}X'.$$

The next thing to note is that $\Omega_{rR}\Omega_{RR}^{-1}$ follows from a regression of $\varepsilon_{r,t+1}$ on $\varepsilon_{R,t+1}$. Using that the OLS-residuals e_r and e_R are given by

$$\begin{aligned} e_r &= r - X(X'X)^{-1}X'r = M'r. \\ e_R &= R - X(X'X)^{-1}X'R = M'R, \end{aligned}$$

this immediately suggests that an estimate of $\Omega_{rR}\Omega_{RR}^{-1}$ can be obtained from

$$(e_r' e_R)(e_R' e_R)^{-1} = (r'MM'R)(R'MM'R)^{-1} = (r'MR)(R'MR)^{-1}.$$

Using partitioned inverses, we can write for the inverse of $(X\ R)'(X\ R)$:

$$\begin{aligned} &(\begin{pmatrix} X & R \end{pmatrix}' \begin{pmatrix} X & R \end{pmatrix}) \\ = &\begin{pmatrix} (X'X)^{-1} + (X'X)^{-1}X'R(R'MR)^{-1}R'X(X'X)^{-1} & -(X'X)^{-1}X'R(R'MR)^{-1} \\ -(R'MR)^{-1}R'X(X'X)^{-1} & (R'MR)^{-1} \end{pmatrix}. \end{aligned}$$

The OLS estimates of γ and δ can now be written as

$$\begin{aligned}\widehat{\gamma} &= (X'X)^{-1}X'r + (X'X)^{-1}X'R(R'MR)^{-1}R'X(X'X)^{-1}X'r - (X'X)^{-1}X'R(R'MR)^{-1}R'r \\ &= \widehat{\gamma}'_r + \widehat{\gamma}'_R\{(R'MR)^{-1}R'X(X'X)^{-1}X'r - (R'MR)^{-1}R'r\} \\ &= \widehat{\gamma}'_r + \widehat{\gamma}'_R\{(R'MR)^{-1}R'Mr\} = (\widehat{\gamma}'_r - \widehat{\gamma}'_R\widehat{\Omega}^{-1}_{RR}\widehat{\Omega}_{Rr}),\end{aligned}$$

and

$$\begin{aligned}\widehat{\delta} &= (R'MR)^{-1}R'r - (R'MR)^{-1}R'X(X'X)^{-1}X'r \\ &= (R'MR)^{-1}\{R'r - R'X(X'X)^{-1}X'r\} \\ &= (R'MR)^{-1}R'Mr = \widehat{\Omega}^{-1}_{RR}\widehat{\Omega}_{Rr},\end{aligned}$$

which is what we wanted to show.

withe $E[\varepsilon_{R,t+1}] = E[\varepsilon_{R,t+1}Z_t] = 0$, $E[\varepsilon_{r,t+1}] = E[\varepsilon_{r,t+1}Z_t] = 0$, and $\varepsilon_{R,t+1}$ and $\varepsilon_{r,t+1}$ jointly *i.i.d.* with variances and covariances given by Ω_{RR}, Ω_{rr}, and Ω_{rR}. It follows that in the regression

$$r_{t+1} = \gamma Z_t + \delta R_{t+1} + u_{t+1},$$

with $E[u_{t+1}] = 0$, $E[u_{t+1}Z_t] = 0$, and $E[u_{t+1}R_{t+1}] = 0$, the OLS-estimates $\widehat{\gamma}$ and $\widehat{\delta}$ are consistent for

$$\gamma = (\gamma'_r - \gamma'_R\Omega^{-1}_{RR}\Omega_{Rr}) \text{ and } \delta = \Omega^{-1}_{RR}\Omega_{Rr}.$$

To see this, first rewrite the regression model as

$$r = \begin{pmatrix} Z & R \end{pmatrix}\begin{pmatrix} \gamma \\ \delta \end{pmatrix} + u,$$

with r a $T \times N$ matrix, Z a $T \times (L+1)$ matrix, R a $T \times K$ matrix, γ a $(L+1) \times N$ matrix, δ a $K \times N$ matrix, and u a $T \times N$ matrix of error terms. Define the idempotent matrix M as

$$M = I_T - Z(Z'Z)^{-1}Z'.$$

The next thing to note is that $\Omega_{rR}\Omega^{-1}_{RR}$ follows from a regression of $\varepsilon_{r,t+1}$ on $\varepsilon_{R,t+1}$. Using that the OLS-residuals e_r and e_R are given by

$$\begin{aligned}e_r &= r - Z(Z'Z)^{-1}Z'r = M'r. \\ e_R &= R - Z(Z'Z)^{-1}Z'R = M'R,\end{aligned}$$

this immediately suggests that an estimate of $\Omega_{rR}\Omega^{-1}_{RR}$ can be obtained from

$$(e'_r e_R)(e'_R e_R)^{-1} = (r'MM'R)(R'MM'R)^{-1} = (r'MR)(R'MR)^{-1}.$$

Using partitioned inverses, we can write for the inverse of $(Z\ R)'(Z\ R)$:

$$
\begin{aligned}
&((\,Z \quad R\,)'(\,Z \quad R\,)) \\
= &\begin{pmatrix} (Z'Z)^{-1} + (Z'Z)^{-1}Z'R(R'MR)^{-1}R'Z(Z'Z)^{-1} & -(Z'Z)^{-1}Z'R(R'MR)^{-1} \\ -(R'MR)^{-1}R'Z(Z'Z)^{-1} & (R'MR)^{-1} \end{pmatrix}.
\end{aligned}
$$

The OLS estimates of γ and δ can now be written as

$$
\begin{aligned}
\widehat{\gamma} &= (Z'Z)^{-1}Z'r + (Z'Z)^{-1}Z'R(R'MR)^{-1}R'Z(Z'Z)^{-1}Z'r - (Z'Z)^{-1}Z'R(R'MR)^{-1}R'r \\
&= \widehat{\gamma}_r' + \widehat{\gamma}_R'\{(R'MR)^{-1}R'Z(Z'Z)^{-1}Z'r - (R'MR)^{-1}R'r\} \\
&= \widehat{\gamma}_r' + \widehat{\gamma}_R'\{(R'MR)^{-1}R'Mr\} = (\widehat{\gamma}_r' - \widehat{\gamma}_R'\widehat{\Omega}_{RR}^{-1}\widehat{\Omega}_{Rr}),
\end{aligned}
$$

and

$$
\begin{aligned}
\widehat{\delta} &= (R'MR)^{-1}R'r - (R'MR)^{-1}R'X(X'X)^{-1}X'r \\
&= (R'MR)^{-1}\{R'r - R'X(X'X)^{-1}X'r\} \\
&= (R'MR)^{-1}R'Mr = \widehat{\Omega}_{RR}^{-1}\widehat{\Omega}_{Rr},
\end{aligned}
$$

which is what we wanted to show.

Appendix 2.B The spanning test-statistic in terms of Sharpe ratios

In this appendix we show how the spanning test statistic can be interpreted in terms of Sharpe ratios, a result that was presented in Section 2.5.3. Recall from Section 2.5.3 that the covariance matrix of the OLS-estimates $\widehat{b}$ equals

$$\Sigma_{\varepsilon\varepsilon} \otimes T^{-1} \begin{pmatrix} 1 + \mu_R' \Sigma_{RR}^{-1} \mu_R & -\mu_R' \Sigma_{RR}^{-1} \\ -\Sigma_{RR}^{-1} \mu_R & \Sigma_{RR}^{-1} \end{pmatrix}.$$

Premultiplying with H_{span} and postmultiplying with H_{span} as defined in (26) yields

$$\begin{aligned} & H_{span} \left(\Sigma_{\varepsilon\varepsilon} \otimes T^{-1} \begin{pmatrix} 1 + \mu_R' \Sigma_{RR}^{-1} \mu_R & -\mu_R' \Sigma_{RR}^{-1} \\ -\Sigma_{RR}^{-1} \mu_R & \Sigma_{RR}^{-1} \end{pmatrix} \right) H_{span}' \\ = \; & \Sigma_{\varepsilon\varepsilon} \otimes T^{-1} \begin{pmatrix} 1 + C_R & -B_R \\ -B_R & A_R \end{pmatrix}, \end{aligned} \qquad \text{(B.1)}$$

the inverse of which is

$$\Sigma_{\varepsilon\varepsilon}^{-1} \otimes \frac{T}{A_R(1+C_R) - B_R^2} \begin{pmatrix} A_R & B_R \\ B_R & 1 + C_R \end{pmatrix}. \qquad \text{(B.2)}$$

Similarly, for h_{span} in (26) we have

$$\left(I_N \otimes \begin{pmatrix} 1 & 0_K' \\ 0 & \iota_K' \end{pmatrix} \right) b - I_N \otimes \begin{pmatrix} 0 \\ \iota \end{pmatrix} = \begin{pmatrix} \alpha_1 \\ \beta_1 \iota_K - 1 \\ \alpha_2 \\ \beta_2 \iota_K - 1 \\ \vdots \\ \alpha_N \\ \beta_N \iota_K - 1 \end{pmatrix}. \qquad \text{(B.3)}$$

Premultiplying (B.2) with h_{span} and postmultiplying with h_{span}', we get, after replacing population moments by their sample equivalents:

$$\xi_W^{span} = T \frac{\widehat{A}_R \widehat{\alpha}' \widehat{\Sigma}_{\varepsilon\varepsilon}^{-1} \widehat{\alpha} - 2\widehat{B}_R \widehat{\alpha}' \widehat{\Sigma}_{\varepsilon\varepsilon}^{-1} (\iota_N - \widehat{\beta}\iota_K) + (1 + \widehat{C}_R)(\iota_N - \widehat{\beta}\iota_K)' \widehat{\Sigma}_{\varepsilon\varepsilon}^{-1} (\iota_N - \widehat{\beta}\iota_K)}{\widehat{A}_R(1 + \widehat{C}_R) - \widehat{B}_R^2}. \qquad \text{(B.4)}$$

Next note that the maximum attainable Sharpe ratio from R_{t+1}, for $\eta = B_R/A_R$, is equal to

$$\theta_R \left(\frac{B_R}{A_R} \right)^2 = C_R - \frac{B_R^2}{A_R}.$$

For simplicity, write $A = A_R + \Delta A$, $B = B_R + \Delta B$, and $C = C_R + \Delta C$, where the definitions of ΔA, ΔB, and ΔC follow from (2.53) and (2.54). Evaluating $\theta(\eta)$ in this same value of η, we get

$$\theta \left(\frac{B_R}{A_R} \right)^2 \;=\; C - 2B\frac{B_R}{A_R} + A\frac{B_R^2}{A_R^2}$$

$$
\begin{aligned}
&= C_R + \Delta C - 2(B_R + \Delta B)\frac{B_R}{A_R} + (A_R + \Delta A)\frac{B_R^2}{A_R^2} \\
&= \theta_R \left(\frac{B_R}{A_R}\right)^2 + \frac{1}{A_R}\left(A_R \Delta C - 2B_R \Delta B + \frac{B_R^2}{A_R}\Delta A\right)
\end{aligned}
$$

Dividing by $(1 + C_R) - B_R^2/A_R = 1 + \theta_R \left(\frac{B_R}{A_R}\right)^2$ gives

$$
\begin{aligned}
\frac{\theta\left(\frac{B_R}{A_R}\right)^2 - \theta_R\left(\frac{B_R}{A_R}\right)^2}{1 + \theta_R\left(\frac{B_R}{A_R}\right)^2} &= \frac{A_R \Delta C - 2B_R \Delta B + \frac{B_R^2}{A_R}\Delta A}{A_R(1 + C_R) - B_R^2} \\
&= \frac{A_R \Delta C - 2B_R \Delta B + \left(C_R + 1 - 1 - \theta\left(\frac{B_R}{A_R}\right)^2\right)\Delta A}{A_R(1 + C_R) - B_R^2} \\
&= \frac{A_R \Delta C - 2B_R \Delta B + (1 + C_R)\Delta A}{A_R(1 + C_R) - B_R^2} - \frac{\Delta A}{A_R}.
\end{aligned}
$$

Replacing all population moments with their sample equivalents again and noting that $1/A_R$ is the variance of the global minimum variance portfolio of R_{t+1}, i.e., $1/A_R = (\sigma_R^0)^2$, and similarly, $1/A = (\sigma^0)2$, we finally obtain

$$
\begin{aligned}
\xi_W^{span} &= T\frac{\widehat{\theta}\left(\frac{B_R}{A_R}\right)^2 - \widehat{\theta}_R\left(\frac{B_R}{A_R}\right)^2}{1 + \widehat{\theta}_R\left(\frac{B_R}{A_R}\right)^2} + T\frac{\widehat{A} - \widehat{A}_R}{\widehat{A}_R} \\
&= T\left(\frac{1 + \widehat{\theta}(\eta_R^0)^2}{1 + \widehat{\theta}_R(\eta_R^0)^2} + \frac{(\widehat{\sigma}_R^0)^2}{(\widehat{\sigma}^0)^2} - 2\right).
\end{aligned}
$$

Appendix 2.C The limiting distribution of the estimated specification error bound

In this appendix we show how the limiting distribution of the estimated specification error bound $\widehat{\delta}$ presented in Section 2.6.1, can be derived from the assumptions made in Section 2.6.2 (see also Hansen, Heaton & Luttmer (1995)). Recall from Section 2.6.2 that $\phi(\lambda)_t$ is defined as

$$\phi(\lambda)_t \equiv y_t^2 - (y_t - \lambda'(R_{t+1} - \mu_R))^2 - 2\lambda'(\iota_K - v\mu_R). \tag{C.1}$$

The maximizer of $E[\phi(\lambda)_t]$ is λ_0 and the maximizer of $T^{-1}\Sigma_t\phi(\lambda)_t$ is $\widehat{\lambda}$, so

$$\delta^2 = E[\phi(\lambda_0)_t], \text{ and } \widehat{\delta}^2 = T^{-1}\Sigma_t\phi(\widehat{\lambda})_t.$$

Also recall that Hansen, Heaton, & Luttmer (1995) assume that

$$\frac{1}{\sqrt{T}}\begin{pmatrix} \Sigma_t\{\phi(\lambda_0)_t - E[\phi(\lambda_0)_t]\} \\ \Sigma_t\{(\widetilde{m}(v)_t R_t - \iota_k) - E[\widetilde{m}(v)_t R_t - \iota_k]\} \end{pmatrix}$$

converges in distribution to a normally distributed random vector with mean zero and covariance matrix V, the $(1,1)$-element of which is denoted by $\sigma(\phi)^2$. It is now straightforward to derive the limiting distribution of $\widehat{\delta}$ by using the decomposition

$$\sqrt{T}(\widehat{\delta}^2 - \delta^2) = \frac{1}{\sqrt{T}}\Sigma_t\{\phi(\widehat{\lambda})_t - \phi(\lambda_0)_t\} + \frac{1}{\sqrt{T}}\Sigma_t\{\phi(\lambda_0) - E[\phi(\lambda_0)].$$

the first term of which converges to zero, (see Hansen, Heaton, & Luttmer (1995)) and the second term of which is assumed to converge in distribution to $N(0, \sigma\phi^2)$. Using a Taylor series expansion for $\sqrt{T}\widehat{\delta}^2$ gives

$$\begin{aligned} \sqrt{T}\widehat{\delta}^2 &\approx \sqrt{T}\delta^2 + 2\delta\sqrt{T}(\widehat{\delta} - \delta) \Rightarrow \\ & \sqrt{T}(\widehat{\delta} - \delta) \xrightarrow{d} N(0, \frac{\sigma(\phi)^2}{4\delta^2}). \end{aligned}$$

Chapter 3

Testing for Spanning with Futures Contracts and Nontraded Assets: A General Approach

3.1 Introduction

The central question in the first part of this thesis is whether investors will value a richer investment opportunity set that results from adding securities to the assets that are already in their portfolio. As outlined in the previous chapter, an investor with a mean-variance utility function is indifferent with respect to holding the additional securities if the Minimum-Variance Frontier (MVF) of the set of assets in an investors portfolio coincides with the MVF of the extended set of these same assets plus the additional securities, in which case there is *mean-variance spanning*. If the two MVF's only have one point in common then there is *intersection*, and only investors for whom the intersection portfolio is optimal need not invest in the additional securities. From Huberman & Kandel (1987) and Chapter 2 it is well known how regression analysis can be used to test for mean-variance spanning. The aim of this chapter is to provide a regression framework in which it is straightforward to test for mean-variance spanning as well as spanning for arbitrary classes of other utility functions. Within this regression framework, we also show how to test for spanning in case of futures contracts and when there are nontraded assets. An example of a nontraded asset is the position in a foreign currency of an exporter. Other examples are given by a pension fund or insurance company that does not want to trade its liabilities, or farmers that cannot trade their harvest nor equity claims to their corporation. Hirshleifer (1988a) for instance, points out that many agricultural assets can not be traded costlessly and stresses that equity claims to growers and handlers of agricultural assets are difficult to issue because of moral hazard and adverse selection problems. From the literature on futures markets it is well known that nontraded positions are important determinants of the agents portfolio choice as well as of the equilibrium expected futures returns (see, e.g., Stoll (1979), Hirshleifer (1988a, 1988b, 1989) and Chapter 6 of this thesis). In general, adding a security or a set of securities to a given set of assets may well be beneficial to some investors but not to others, depending on their utility function and the nontraded assets in their current portfolio. Apart from the presence of nontraded assets, we will assume that there are no market frictions such as short sales restrictions or transaction costs. Tests for spanning in economies with such market frictions are discussed in Chapter 4.

We show that allowing for non mean-variance utility functions, testing for spanning implies that alternative regression models have to be considered in which restrictions similar to the ones derived in the previous chapter have to be tested. For the case in which an investor has a position in a non-traded asset, the payoff of his portfolio will change because of this position. For such investors, regression based tests for spanning can be performed by using returns that are adjusted for the position in the non-traded asset. Finally, for spanning tests the crucial difference between futures contracts and assets is that futures are zero-investment securities. We show that this implies that the restrictions on the regression coefficients imposed by spanning have to be modified to reflect the zero-investment property of futures contracts.

The regression framework for spanning tests proposed in this chapter provides clear interpretations in terms of performance measurement, similar to the results in Chen & Knez (1996) and Cumby & Glen (1990) for instance. However, a characteristic of the empirical applications of performance evaluation in the literature so far is that they are tests for intersection rather than spanning. Thus, performance is usually measured with respect to an optimal benchmark portfolio for a specific utility function. Our regression framework allows us to test for spanning for a set of utility functions simultaneously. The regression results then provide the additional advantage that we can evaluate the performance relative to a specific utility function while controlling for the performance relative to other utility functions.

When applied to a basic set of assets, consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) indices, it appears that unconditional tests reject the null hypothesis of spanning for many futures contracts, depending on whether we test for mean-variance, logarithmic utility, or power utility spanning. When there is a nonmarketable position in a particular commodity, the null hypothesis of spanning is easily rejected for the futures contract that has the exposure asset as the underlying value for all utility functions considered. When there is an exposure to a foreign currency, spanning can only be rejected for investors with a power utility function that reflects a preference for skewness. Thus, for our benchmark indices, currency futures only show an abnormal performance for a power utility function but not for mean-variance and logarithmic utility functions. Finally, when net futures positions of large traders are used to predict futures returns, conditional tests reject the null hypothesis of spanning for many futures contracts for all utility functions considered.

The plan of this chapter is as follows. In Section 3.2 we will discuss the notion of spanning for arbitrary utility functions and show how to test for spanning. In Section 3.3 the tests for spanning will be extended to the case where there are futures contracts and nontraded assets. Here we also show how conditional information can be incorporated in tests for spanning. In Section 3.4 an illustration of the tests is presented for a set of commodity and currency futures contracts, with different kinds of exposures. Finally, Section 3.5 contains a summary and some concluding remarks.

3.2 Testing for spanning

3.2.1 Spanning for arbitrary classes of utility functions

The initial setup in this chapter is similar to the setup in Chapter 2. To repeat, suppose that an investor initially considers a set of K assets, the gross returns of which are given by the vector R_{t+1}. The set R_{t+1} may or may not contain a risk free asset. Throughout this section we will take the case where R_{t+1} consists of non-zero investment securities only. The case of zero-investment securities, such as futures and forward contracts, will be considered in the next section. Assuming that there are no market frictions such as short sales constraints and transaction costs and that the law of one price holds, we know that there exists a stochastic discount factor or pricing kernel, M_{t+1}, such that:

$$E[M_{t+1}R_{t+1} \mid I_t] = \iota_K, \tag{3.1}$$

where ι_K is a K-dimensional vector containing ones, and I_t is the information set that is known to the investor at time t.

Recall that M_{t+1} in (3.1) can be derived from the first order conditions of a discrete time intertemporal portfolio selection problem. Usually, this optimization program is solved using dynamic programming (see, e.g., Ingersoll (1987) and Duffie (1988)). The pricing kernel M_{t+1} is then known to be proportional to the derivative of the derived utility function of wealth (or the value function in the dynamic program), given the agent's optimal portfolio choice. Suppose that the agent subsequently also takes additional securities with gross return r_{t+1} into account when optimizing his utility. For notational convenience we will assume that r_{t+1} contains only one element. Spanning occurs if the original first order conditions for the optimal portfolio choice in (3.1) are also satisfied by the additional security r_{t+1}, i.e., if:

$$E[M_{t+1}r_{t+1} \mid I_t] = 1. \tag{3.2}$$

In other words, spanning occurs for a class of agents if they will not be able to increase their utility by incorporating r_{t+1} in their portfolio. Since each pricing kernel M_{t+1} refers to different preferences of an agent, spanning for a given class of preferences occurs if the above reasoning holds for the set of pricing kernels associated with this class of preferences. For instance, mean-variance spanning holds if all pricing kernels that satisfy (3.1) and that are linear in the returns R_{t+1}, also satisfy (3.2). Therefore, we explicitly mention a particular set of pricing kernels $\mathcal{M}$ in the following definition.

Definition 1 *Let $\mathcal{M}$ be a set of pricing kernels for the assets R_{t+1}, i.e.*

$$\forall M_{t+1} \in \mathcal{M} : E[M_{t+1}R_{t+1} \mid I_t] = \iota_K.$$

Then $\mathcal{M}$-spanning of the asset r_{t+1} by the assets R_{t+1} holds by definition if the following condition is satisfied:

$$H' : \forall M_{t+1} \in \mathcal{M} : E[M_{t+1}r_{t+1} \mid I_t] = 1.$$

The case where $\mathcal{M}$ is a singleton is denominated intersection.

This definition clearly generalizes the definition of mean-variance spanning in Huberman & Kandel (1987)[7].

If the asset set R_{t+1} does not span the asset r_{t+1}, then $\lambda_t = E[M_{t+1}r_{t+1} \mid I_t] - 1$ can be interpreted as a performance measure for the asset r_{t+1} relative to the set R_{t+1} (see, e.g., Chen & Knez (1996) and Glosten & Jagannathan (1994)). If $\lambda_t > (<)0$ for a given M_{t+1}, then an investor with a utility function that corresponds to M_{t+1} can improve his portfolio performance by taking a long (short) position in the asset r_{t+1} in addition to his investments in the set R_{t+1}. For instance, the results in Cumby & Glen (1990) indicate that a sample of fifteen U.S.-based internationally diversified mutual funds do not have added value for investors with either quadratic or power utility functions[8] relative to a broad international equity index and a risk free asset.

One characteristic of the empirical applications of performance evaluation such as in Cumby & Glen (1990) and Chen & Knez (1996), is that the tests are for intersection rather than spanning. This implies that performance is measured with respect to the optimal portfolio of a specific utility function[9]. Our methodology tests for spanning for a prespecified *set* of utility functions and a given set of assets. In order to test for spanning, we need an equivalent formulation of H' that can be tested easily using regression.

[7] The definition for spanning used in this paper originates from Huberman & Kandel (1987). Note however, that this definition is slightly different from (although closely related to) another definition that is used in the literature and that can be found, for instance, in Ross (1978). Our results can be interpreted as an extension of Ross (1978) to the case where $\mathcal{M}$ is a subset of all monotone concave utility functions.

[8] Cumby & Glen test the performance of the funds using both Jensen's α relative to a mean-variance efficient portfolio and a positive weighting measure as proposed by Grinblatt & Titman (1989) with weights that are equal to the derivative of a power utility function evaluated at the optimal portfolio choice.

[9] Performance is measured with respect to a specific utility function because no specific assumption is made about the distribution of asset returns. If asset returns were jointly normally distributed for instance, a test for mean-variance intersection would imply performance measurement that is independent of the specific utility function.

3.2.2 Testing for spanning

To obtain an easily testable equivalent formulation of the spanning hypothesis H', we need the following notation. Write $W = \{w \in I\!R^k : w'\iota_K = 1\}$, so that W consists of those portfolio choices that are valid for an agent in the sense that the portfolio weights of assets sum to one. We can now state our main result.

Proposition 1 *Let $\mathcal{M}$ be a set of kernels that includes at least the minimum second moment pricing kernel $M^0_{t+1} = \iota'_K E_t[R_{t+1}R'_{t+1}]^{-1}R_{t+1}$*[10]*. Then the asset r_{t+1} is $\mathcal{M}$-spanned by the assets R_{t+1} if and only if*[11]

$$\mathrm{Proj}(r_{t+1} \mid \mathcal{M} \cup \{w'R_{t+1} : w \in W\}) = w'R_{t+1}, \text{ for some } w \in W. \tag{3.3}$$

In Appendix 3.A we prove a generalized version of this proposition, in which we also allow for zero-investment securities. The result in Proposition 1 states that r_{t+1} can be written as a portfolio w of the assets R_{t+1} and an error term that is orthogonal to the set $\mathcal{M}$ of pricing kernels under consideration. This implies that all agents with utility functions corresponding to the class $\mathcal{M}$ prefer $w'R_{t+1}$ over r_{t+1}, since they do not value the return difference $r_{t+1} - w'R_{t+1}$. If r_{t+1} is not $\mathcal{M}$-spanned by R_{t+1}, then r_{t+1} is of value to some investors because the difference between r_{t+1} and a portfolio of the initial assets R_{t+1} covaries with their intertemporal marginal rate of substitution.

The strength of Proposition 1 is that testing H' is straightforward once the projection in (3.3) is available. If we denote this projection by $\hat{r}_{t+1}$, i.e.,

$$\hat{r}_{t+1} = \mathrm{Proj}\left(r_{t+1} \mid \mathcal{M} \cup \{w'R_{t+1} : w \in W\}\right), \tag{3.4}$$

then $\hat{r}_{t+1}$ follows from a regression. After $\hat{r}_{t+1}$ is determined, testing H' can be done by testing the hypothesis

$$H : \quad \hat{r}_{t+1} = w'R_{t+1}, \text{ for some } w \in W \tag{3.5}$$

In order to calculate the projection in (3.4) a functional form for the kernels in $\mathcal{M}$ is needed. It is well known that the pricing kernel is proportional to the marginal utility of consumption, given the optimal portfolio and consumption choice of the agent. The envelope theorem in turn implies that the pricing kernel is also proportional to the marginal derived utility of wealth (see, e.g., Ingersoll

[10] Notice that the minimum second moment kernel is automatically included in $\mathcal{M}$ in all extensions of mean-variance spanning.

[11] As usual we work in the Hilbert space of square integrable random variables. Projection in this space will be denoted by Proj.

(1987)). This allows us to derive the projection in (3.4) from a regression of r_{t+1} on (functions of) the initial asset returns R_{t+1} only.

The case of mean-variance spanning is discussed extensively in Chapter 2. However, it is useful to put these ideas in our more general context. As outlined in Chapter 2, the pricing kernels associated with mean-variance optimizing behaviour are linear in the returns R_{t+1}. The set $\mathcal{M}$ therefore coincides with the set of pricing kernels that traces out the volatility bound in Hansen & Jagannathan (1991). More precisely, for investors that choose their portfolio from the assets R_{t+1} the set $\mathcal{M}$ is in this case given by all kernels of the form

$$M_{t+1}(v) = v + \alpha'(R_{t+1} - E_t[R_{t+1}]),\ v \in I\!R, \tag{3.6}$$

where

$$\alpha = Var_t[R_{t+1}]^{-1}\{\iota_K - vE_t[R_{t+1}]\}.$$

Therefore, $\hat{r}_{t+1}$ equals the projection of r_{t+1} on all stochastic variables of the form $\alpha_0 + \alpha' R_{t+1}$. Consequently, assuming that all expected returns and (co)variances of the returns do not vary over time (this assumption will be relaxed in Section 3.3.3), $\hat{r}_{t+1}$ can be obtained from the following regression

$$r_{t+1} = \alpha + \beta' R_{t+1} + \varepsilon_{t+1}. \tag{3.7}$$

The hypothesis H now becomes

$$H: \quad \alpha = 0 \text{ and } \beta'\iota_K = 1.$$

These linear restrictions are of course identical to the ones given in Chapter 2 and are straightforward to test using a Wald test.

If the set R_{t+1} also spans r_{t+1} for investors with non mean-variance (derived) utility function(s) $U(w^{*\prime}R_{t+1})$, where w^* denotes the optimal portfolio choice for the investor, then the projection in (3.3) implies that the error term ε_{t+1} in (3.7) should be orthogonal to the marginal derived utility $U'(w^{*\prime}R_{t+1})$. To test for both mean-variance spanning and spanning for N different utility function(s) $U(w^{*\prime}R_{t+1})$, given knowledge of w^*, the projection of r_{t+1} can now be obtained from the regression

$$r_{t+1} = \alpha + \beta' R_{t+1} + \sum_{i=1}^{N} \gamma_i U_i'(\varphi_i^{*\prime} R_{t+1}) + \varepsilon_{t+1}, \tag{3.8}$$

where $U_i'(\varphi_i^{*\prime} R_{t+1})$, $i = 1, 2, .., N$ are the derivatives of the (non mean-variance) utility functions of interest, i.e., for all utility functions that correspond to kernels in $\mathcal{M}$. The coefficients φ_i^* are proportional to the optimal portfolio weights w_i^* where the constant of proportionality is chosen such that $E[U_i'(\varphi_i^{*\prime} R_{t+1})R_{t+1}] = \iota_K$.

The null hypothesis that there is $\mathcal{M}$-spanning is now equivalent to

$$H:\ \beta'\iota_K = 1,\ \alpha = \gamma_i = 0,\ i = 1, 2, ..., N.$$

As with mean-variance spanning, these restrictions are easy to test using a Wald test.

Given a specific utility function, from the first order conditions

$$E[U'(\varphi^{*\prime} R_{t+1})R_{t+1}] = \iota_K,$$

estimates of φ^*, $\widehat{\varphi}^*$, can be obtained using, for instance, a GMM-estimator. Since in empirical applications $U'(\widehat{\varphi}^{*\prime} R_{t+1})$ in (3.8) is based on the estimated coefficients $\widehat{\varphi}^*$, this will obviously affect the limit distribution of the regression parameters in (3.8). In Appendix 3.B the limit distribution of the regression parameters is derived, accounting for the fact that we have to estimate φ.

Testing whether there is spanning for the utility functions $U_i(\varphi_i^{*\prime} R_{t+1})$ can also be done in a GMM-framework by testing the overidentifying restrictions that $E[U_i'(\varphi_i^{*\prime} R_{t+1})r_{t+1}] = \iota_N$ for all i (see, e.g., Chen & Knez (1996)). The advantage of our regression framework is that the regression coefficients in (3.8) have a clear interpretation in terms of optimal portfolio weights and that we can easily test whether there is spanning for a particular utility function while controlling for other utility functions. In case of mean-variance spanning for instance, the optimal portfolio weights for the new assets r_{t+1} can be obtained from the regression parameters in (3.7). For a given value of v, the vector with generalized Jensen measures is equal to $\alpha_J(v) = \alpha + (\beta\iota_K - \iota_N)/v$. Denoting the covariance matrix of the error terms in (3.7) as $\Sigma_{\varepsilon\varepsilon}$, it was already shown in Chapter 2 that the optimal portfolio weights for r_{t+1}, w_r, are proportional to $\Sigma_{\varepsilon\varepsilon}^{-1}\alpha_J(v)$. Note that positive alpha's with respect to the benchmark assets do not necessarily imply long positions in the new efficient portfolio if a number of assets is added to the portfolio of benchmark assets simultaneously. For non mean-variance utility functions, it is in general not possible to give an explicit expression of the new optimal portfolio weights w_r in terms of the regression parameters in (3.8). However, recall that the sign of $\lambda_r = E[M_{t+1}r_{t+1}] - 1$ indicates whether an investor with a utility function that corresponds to M_{t+1} can improve the performance of his portfolio by taking long or short positions in r_{t+1}. In Appendix 3.C it is shown that λ_r can be written as

$$\lambda_r = v\alpha_J(v) + \gamma Var[M_{t+1} - \widehat{M}_{t+1}],$$

where γ is the slope parameter corresponding to M_{t+1} in (3.8), $v = E[M_{t+1}]$, and $\widehat{M}_{t+1}$ =Proj$[M_{t+1} \mid 1, R_{t+1}]$. The parameter γ determines the part of λ_r that is not attributable to Jensen's alpha but that is specific to the utility function that corresponds to M_{t+1}. The slope coefficient γ therefore measures the performance of r_{t+1} relative to the benchmark securities R_{t+1}, after controlling for Jensen's alpha.

Several tests for performance evaluation that are known in the literature can also be interpreted in terms of the framework presented here. For instance, Cumby & Glen (1990) test the performance of international mutual funds for a mean-variance investor and for an investor with a power utility function. Since they use prespecified benchmark portfolios that are optimal for a mean-variance investor and a power-utility investor respectively, their tests can be interpreted as tests for intersection for these two utility functions. In terms of the regression in (3.8) we could (simultaneously) test for mean-variance and power-utility spanning by choosing for $U(w^{*\prime}R_{t+1})$ the specific power utility function that is used by Cumby & Glen (1990).

Similarly, Glosten & Jagannathan (1994) propose a performance test where the set R_{t+1} consists of one asset, an index portfolio, and that is based on a polynomial fit of r_{t+1} on R_{t+1}. In terms of (3.8) this is similar to choosing marginal utilities of the form $U'(R_{t+1}) = R_{t+1}^i$, $i = 2, 3, \ldots$ Our motivation for using a polynomial approach is entirely different from that of Glosten & Jagannathan however. For instance, Glosten & Jagannathan motivate the use of a second order polynomial, i.e. only $U'(R_{t+1}) = R_{t+1}^2$ is used in (3.8), to account for market timing, as also suggested by Treynor & Mazuy (1966). In our framework on the other hand, a quadratic term captures a preference for skewness.

In the empirical application in Section 3.4 we will test for mean-variance spanning and for power-utility spanning. For the power-utility function we will use both a risk aversion coefficient of 0, that corresponds to a logarithmic utility investor, and a risk aversion coefficient of -3, that corresponds to an investor who has a preference for skewness.

3.3 Testing for spanning with futures contracts and non-traded assets

3.3.1 Futures contracts

The main result of the previous section, as stated in Proposition 1, is that spanning of an asset r_{t+1} by a base set of assets R_{t+1} is equivalent to stating that the projection of r_{t+1} on all portfolios of R_{t+1}, $w'R_{t+1}$, and the relevant class of pricing kernels $\mathcal{M}$, gives a portfolio of the base securities. The intuition behind this result is that if r_{t+1} is spanned by the set R_{t+1}, then r_{t+1} can be written as the payoff of a portfolio of the initial assets R_{t+1} plus an idiosyncratic error term that is orthogonal to the asset returns R_{t+1} and the pricing kernels in $\mathcal{M}$ and hence not valued by any of the investors under consideration.

The crucial difference between assets and futures contracts in this respect is that futures contracts do not require any initial investment. Whereas the payoff of an asset at time $t+1$ is its price S_{t+1} (ignoring dividends and the like), the payoff of a futures contract is given by the *change* in the futures price, $F_{t+1} - F_t$. Whereas asset returns are defined as $R_{S,t+1} = \frac{S_{t+1}}{S_t}$, we define a futures return[12] as $R_{F,t+1} = \frac{F_{t+1}-F_t}{F_t}$. In case of futures contracts, Equation (3.1) changes to:

$$E[M_{t+1}R_{F,t+1} \mid I_t] = 0. \tag{3.9}$$

Denote R_{t+1} now as the K-dimensional vector, the first K_S elements of which are asset returns, $R_{S,t+1}$, and the last K_F elements of which are futures returns, $R_{F,t+1}$, $K = K_S + K_F$. These are the K securities initially considered by the investors. Next, let e_K be a vector consisting of K_S ones and K_F zero's, $e'_K = (\iota'_{K_S}\ 0'_{K_F})$. We can now generalize Equations (3.1) and (3.9) to:

$$E[M_{t+1}R_{t+1} \mid I_t] = e_K. \tag{3.10}$$

Finally, write $W_e^S = \{w \in I\!R^K : \ w'e_K = 1\}$ and $W_e^F = \{w \in I\!R^K : \ w'e_K = 0\}$. Thus, W_e^S and W_e^F define portfolios in which the asset weights must sum to either one ($w \in W_e^S$) or zero ($w \in W_e^F$), but there are no restrictions on the futures positions. Note that the minimum second moment portfolio is now given by $\alpha = E_t[R_{t+1}R'_{t+1}]^{-1}e_K$. If R_{t+1} only contains futures contracts, then $\alpha = 0$. As a generalization of Proposition 1 it is now straightforward to show that r_{t+1} is $\mathcal{M}$-spanned by the securities R_{t+1} if and only if

$$\text{Proj}(r_{t+1} \mid \mathcal{M} \cup \{w'R_{t+1} : w \in \widetilde{W}\}) = w'R_{t+1}, \text{ for some } w \in \widetilde{W}, \tag{3.11}$$

where $\widetilde{W} = W_e^S$ if r_{t+1} refers to a non-zero investment security, and $\widetilde{W} = W_e^F$ if r_{t+1} refers to a zero-investment security. The proof of this proposition is given in Appendix 3.A. Given this proposition, testing for spanning proceeds in the same way as outlined in Section 3.2, except that in case r_{t+1} is a futures contract the restriction that $w'e_K = 1$ has to be replaced by the restriction $w'e_K = 0$.

First of all, note from (3.11) that spanning only imposes restrictions on the sum of the asset weights, but not on the futures positions. This reflects the fact that assets require a non-zero investment, while futures contracts do not require any investment. Second, if r_{t+1} refers to a non-zero investment security, spanning requires that the asset weights in w sum to one, while if r_{t+1} refers to a zero-investment security like futures contracts, the asset weights in w must sum to zero.

12 Actually, the term futures return itself is a misnomer, for the same reason that futures contracts do not require an investment.

If the return on a futures contract is to be written as the return on a portfolio of assets and futures, then this must be a zero-investment portfolio, since the futures contract itself does not require any investment either. Thus, the difference in the restrictions for futures contracts and assets stems from the fact that futures contracts are zero investment securities.

3.3.2 Nontraded Assets

So far we treated all investors as if they had the same investment opportunity set. However, because investors can have positions in nontraded assets, i.e., they can face different *nonmarketable risks*, they may face different investment opportunity sets. For example, the investment opportunity set of an exporter is affected by his exposure to foreign currency. Similarly, the investment opportunity sets of pension funds and insurance companies are affected by their liabilities. Consequently, when considering additional securities, the initial set of assets may span the extended set for one agent, but not for others. The reason is that the presence of a nontraded asset changes the net portfolio payoff for an investor. The effect of nontraded assets on portfolio choice and on expected asset and futures returns, have been analyzed extensively by Stoll (1979) and Hirshleifer (1988a, 1988b, 1989) for the mean-variance case. Hirshleifer notes that especially for farmers nontraded assets are important because equity claims to their companies are difficult to issue. The empirical evidence of so-called hedging pressure on expected futures returns, which reflects the aggregate nontraded positions of agents in the economy, suggests that nontraded assets are indeed important for many agents (see, e.g., Carter, Rausser & Schmitz (1983), Chang (1985), Bessembinder (1992) and Chapter 6 of this thesis.

Let W_t be the wealth invested in assets by an investor, excluding nontraded assets. The fraction of wealth invested in asset j is given by w_{Sj}, and w_S is a vector containing all w_{Sj}. Notice that $w_S'\iota = 1$. Besides investing in assets, an investor can also take a position in futures contract k, which is also expressed as a fraction of W_t. The vector w_F similarly contains all the futures positions of the agent. Finally, the agent may have a position in a nontraded asset with a size q_X that yields a return $R_{X,t+1}$. The size of the position is also expressed as a fraction of W_t, implying that $\iota' w_S + q_X$ will not be equal to one if $q_X \neq 0$. Thus, the total return on his invested wealth for the investor is given by:

$$R_{W,t+1} = w_S' R_{S,t+1} + w_F' R_{F,t+1} + q_X R_{X,t+1}. \tag{3.12}$$

Of course, a similar expression arises when the additional security r_{t+1} is included.

Notice that the asset weights w_{Sj} must sum to one. Therefore, an equivalent way of writing the total return in (3.12) is:

$$R_{W,t+1} = w_S'(R_{S,t+1} + q_X \iota R_{X,t+1}) + w_F' R_{F,t+1} = w_S' \widetilde{R}_{S,t+1} + w_F' R_{F,t+1}, \tag{3.13}$$

where $\widetilde{R}_{S,t+1}$ are the returns adjusted for the position in the nontraded asset. Since there is only a restriction on the asset weights and not on the futures positions, only the asset weights must be adjusted for the position in the nontraded asset. Denote $\widetilde{R}_{t+1}$ as the total adjusted return vector, $\widetilde{R}_{t+1}' = (\widetilde{R}_{S,t+1}' R_{F,t+1}')$. To see the implications of the presence of nontraded assets for spanning, observe that one valid stochastic discount factor is the intertemporal marginal rate of substitution of agent i. Since agent i will choose his portfolio taking into account the nontraded asset, his interest will be in the adjusted returns, $\widetilde{R}_{t+1}$, rather than the normal returns, R_{t+1}. It's easy to see that this implies that the following should hold:

$$E[\widetilde{M}_{t+1} \widetilde{R}_{t+1} \mid I_t] = e_K, \tag{3.14}$$

where $\widetilde{M}_{t+1}$ indicates the stochastic discount factor that prices the adjusted asset returns.

It is now straightforward to test for spanning taking into account the nontraded assets. $\mathcal{M}$-spanning of r_{t+1} by the securities R_{t+1} occurs if and only if

$$\text{Proj}(\widetilde{r}_{t+1} \mid \mathcal{M} \cup \{w' \widetilde{R}_{t+1} : w \in W\}) = w' \widetilde{R}_{t+1}, \text{ for some } w \in W. \tag{3.15}$$

All tests described in Section 3.2 are still valid, provided that we replace the asset returns $R_{S,t+1}$ and $r_{S,t+1}$ by adjusted returns, $\widetilde{R}_{S,t+1}$ and $\widetilde{r}_{S,t+1}$, while the futures returns remain unchanged.

3.3.3 Testing for spanning using conditioning information

So far we assumed that expected returns, (co)variances, and all relevant expected moments are constant over time. Especially in the futures markets literature however, there is substantial evidence of return predictability. For instance, there is ample evidence that futures returns can be predicted from the net positions of large traders, known as hedging pressure (see e.g. Carter, Rausser, & Schmitz (1983), Chang (1985), Bessembinder (1992) and Chapter 6 of this thesis). Also, Fama & French (1987) present evidence that commodity returns can be predicted from the observed spread between the futures and the spot price. Similarly, Glen & Jorion (1993) show that the efficiency of international asset portfolios significantly improves if it is taken into account that expected currency returns depend on the forward premium. Finally, there is substantial evidence that stock and bond returns can be predicted using instruments like lagged returns, dividend yields, short term interest rates, and default premiums (see, e.g., Ferson (1995)).

If expected returns are dependent on conditioning information at time t, then the parameters α and β in equation (3.7) should also be dependent on that information. In this case, there may be spanning in one period, but not in other periods, because of a change in economic conditions.

As in Section 2.4.2, assume that expected returns are linearly dependent on a vector of variables x_t that are in the investor's information set at time t, i.e., $x_t \in I_t$. We will still assume that the (co)variances of the returns are constant. The extension to time-varying covariances is straightforward however. It was shown in Chapter 2 that tests for mean-variance spanning can be based on the following regression:

$$r_{t+1} = \alpha_0 + \alpha' x_t + \beta' R_{t+1} + \varepsilon_{t+1}. \tag{3.16}$$

In this case spanning occurs for arbitrary values of x_t if and only if $\alpha_0 = \alpha = 0$ and $\beta' \iota_K = 1$. In case $\beta' \iota_K = 1$ and $\alpha \neq 0$, it follows that spanning occurs for $\alpha_0 = -\alpha' x_t$. This implies that we can test whether there is spanning under certain economic conditions, i.e., for specific values of x_t. Alternative ways to incorporate conditional information in tests for mean-variance spanning were discussed in Chapter 2.

This way of using conditioning information to allow for return predictability readily extends to the tests for spanning for arbitrary classes of utility functions. We will again assume that only expected returns are dependent on conditioning information x_t that is known at time t. The (co)variances and all other relevant moments are assumed to be constant. As in the unconditional case, if there is also spanning of r_{t+1} by R_{t+1} for investors with non mean-variance (derived) utility functions $U(w^{*\prime} R_{t+1})$, then the error term ε_{t+1} in (3.16) should be orthogonal to the marginal derived utility $U'(w^{*\prime} R_{t+1})$. It is now again straightforward to show that testing for $\mathcal{M}$-spanning can be based on the regression

$$r_{t+1} = \alpha_0 + \alpha' x_t + \beta' R_{t+1} + \sum_{i=1}^{N} \gamma_i U_i'(\varphi_i^{*\prime} R_{t+1}) + \varepsilon_{t+1}. \tag{3.17}$$

If there is spanning regardless of the value of x_t, i.e., regardless of the economic conditions, then $\alpha_0 = \alpha = \gamma = 0$ and $\beta' \iota_K = 1$. If $\beta' \iota_K = 1$ and $\gamma = 0$, then there is only spanning if $\alpha_0 = -\alpha' x_t$, as in the mean-variance case.

3.4 Empirical results for commodity and currency futures

In this section we illustrate the analysis in the previous sections for a number of commodity and currency futures contracts. We test whether a base set of three international stock indices spans the extended set of these same portfolios plus a number of futures contracts. We use semi-monthly data from January 1984 until December 1993 to construct monthly holding returns for the S&P

Table 3.1: Summary statistics of net monthly returns

The table contains summary statistics for semi-monthly observations of net monthly holding period returns, $(P_{t+1} - P_t)/P_t$, over the period January 1984 - December 1993. The total number of observations in this period is 240. Average returns, standard deviations, median, minimum and maximum are all in percentages.

	Average	Median	Stdev	Skew	Kurt-3	Max	Min
Basic assets							
S&P 500	0.59	0.80	4.23	-0.73	4.82	17.1	-22.1
FAZ (Germany)	1.31	1.28	6.51	-0.05	1.23	18.6	-22.8
FTSE (UK)	1.17	0.70	5.91	-0.06	1.30	17.8	-22.5
Futures							
wheat	0.34	0.65	5.48	0.20	0.99	25.1	-15.3
corn	-0.39	-0.79	6.45	2.18	13.86	45.8	-18.6
soybeans	-0.34	-0.45	5.72	0.72	3.47	26.4	-21.6
soybean meal	0.06	-0.80	6.50	1.31	4.34	36.0	-16.6
soybean oil	-0.14	-0.90	7.36	0.50	0.62	24.5	-18.4
live cattle	0.78	0.78	3.71	-0.07	0.69	14.5	-9.9
live hogs	1.07	0.75	6.18	0.31	0.07	21.6	-15.4
Deutsche Mark	0.42	0.21	3.69	-0.01	-0.15	11.8	-10.5
British Pound	0.42	0.22	3.98	0.24	1.66	18.4	-11.3
Japanese Yen	0.55	-0.07	3.43	0.45	0.43	12.1	-7.5

500, the FTSE (UK) and the FAZ (Germany), as well as for a number of commodity and currency futures. The returns on the FTSE and the FAZ used here, are unhedged dollar returns, so we take the perspective of a US-investor. The three indices used here allow a US-investor to form a well-diversified asset portfolio. Summary statistics for monthly holding returns on the three indices and the futures contracts are presented in Table 3.1. The data for the FTSE and the FAZ are obtained from Datastream, while all other data are obtained from the Futures Industry Institute. Because semi-monthly observations of monthly holding returns create an overlapping samples problem, consistent estimates of the relevant covariance matrices are calculated as in Newey & West (1987a). Returns for the futures contracts are always for the nearest-to-delivery contract, excluding observations in the delivery month.

3.4.1 Unconditional tests without nontraded assets

Table 3.2 reports results of tests whether there is spanning for several utility functions, assuming that all relevant moments of the returns are time-invariant. The first column presents results for the null hypothesis that there is mean-variance spanning of the futures contracts by the three international stock indices. The p-values associated with the Wald test-statistics show that this null hypothesis is only rejected for live cattle futures. This suggests that in a market without frictions and using no conditioning information, most futures contracts considered here do not have added value for a US-investor with a mean-variance utility function.

Table 3.2: Results for unconditional spanning

The results in the table are for the null hypothesis that a basic set of assets spans a futures contract. The column "mean-variance" is for the null hypothesis that there is mean-variance spanning, "log" is for the null hypothesis that there is both mean-variance and log-utility spanning, "power" is for the null hypothesis that there is both mean-variance and power utility spanning, where the power utility function has a risk aversion coefficient of -3, and "log+power" is for the null hypothesis that there is spanning for both logarithmic and power utility functions. The columns "p-value" give the p-values associated with the Wald-test for spanning. $\widehat{\gamma}$ is the estimated slope coefficient associated with the log- and power-utility kernels in the regression in (3.8); t-statistics are in brackets. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

	mean-variance	log			power		
	p-value	$\widehat{\gamma}_{\log}$		p-value	$\widehat{\gamma}_{pow}$		p-value
wheat	0.084	-0.281	(-2.00)	0.015	-0.041	(-0.66)	0.121
corn	0.442	0.009	(0.09)	0.646	-0.021	(-0.38)	0.286
soybeans	0.692	-0.165	(-1.92)	0.084	-0.005	(-0.12)	0.857
soyb. meal	0.695	-0.079	(-0.54)	0.490	0.135	(2.30)	0.013
soyb. oil	0.927	-0.355	(-2.40)	0.018	-0.083	(-1.14)	0.493
live cattle	0.015	-0.032	(-0.17)	0.027	-0.057	(-1.52)	0.000
live hogs	0.074	0.117	(0.36)	0.122	-0.041	(-0.53)	0.102
DMark	0.686	0.088	(0.98)	0.371	0.137	(5.84)	0.000
BPound	0.682	0.139	(1.40)	0.167	0.144	(6.40)	0.000
JYen	0.329	0.034	(0.32)	0.481	0.100	(3.70)	0.000

Table 3.2: Results for unconditional spanning (continued)

	log+power				
	$\widehat{\gamma}_{\log}$		$\widehat{\gamma}_{pow}$		p-value
wheat	-0.286	(-1.93)	0.064	(1.28)	0.008
corn	0.024	(0.22)	-0.023	(-0.66)	0.273
soybeans	-0.189	(-1.79)	0.070	(2.03)	0.038
soyb. meal	-0.357	(-2.30)	0.156	(2.80)	0.000
soyb. oil	-0.264	(-1.27)	0.091	(1.32)	0.018
live cattle	0.061	(0.36)	-0.046	(-0.82)	0.000
live hogs	0.221	(0.76)	-0.088	(-0.91)	0.007
DMark	-0.175	(-0.83)	0.064	(0.87)	0.000
BPound	-0.130	(-0.58)	0.047	(0.60)	0.000
JYen	-0.164	(-0.97)	0.062	(1.08)	0.000

The next three columns of Table 3.2 show tests for the hypothesis that, besides mean-variance spanning, there is also spanning for a logarithmic utility investor. As outlined in Section 3.2, the parameters for the kernels that correspond to these utility functions are estimated using a GMM-estimator. The first columns show estimates of the slope parameter γ in (3.8) along with the t-statistics. The p-values in the third column are for a Wald-test of the hypothesis that the three stock indices span the futures contracts for both mean-variance investors and investors with a logarithmic utility function $U(W_{t+1}) = \log(W_{t+1})$. The spanning hypothesis can now be rejected for wheat and soybean oil futures and, again, for live cattle futures. The t-statistics for the estimated parameter $\widehat{\gamma}$ show coefficients that are significantly different from zero for wheat and soybean oil futures, but not for live cattle futures. Thus, the rejection of the spanning hypothesis in case of live cattle futures is due the fact that there is no mean-variance spanning, but there is no specific effect for the logarithmic utility function. For wheat and soybean oil futures on the other hand, the rejection of the spanning hypothesis is due to the specific effect of the logarithmic utility function. Moreover, the negative sign of $\widehat{\gamma}$ for both these futures contracts indicates that investors with a logarithmic utility function can improve the performance of their portfolio by taking short positions in these contracts. For all other futures contracts the spanning hypothesis can not be rejected.

The next three columns of Table 3.2 show similar results for power utility spanning. Here the null hypothesis is that the futures contracts are spanned by the three stock indices for investors with a mean-variance utility function and for investors with a power utility function $U(W_{t+1}) = \frac{1}{c}W_{t+1}^{c}$, where the risk aversion coefficient $c = -3$, reflecting a preference for skewness. The main difference with mean-variance and log utility spanning is that there is now an abnormal performance of the currency futures. Apparently, US-investors with a preference for skewness would like to hedge their currency exposure that arises from their investments abroad. However, the estimates $\widehat{\gamma}$ indicate that US-investors with a power utility function want to take long positions in the currency futures even though they already have an exposure to the foreign currency. This suggests that the speculative demand for the currency futures outweighs the pure hedge demand for power utility investors.

Finally, the last five columns in Table 3.2 show test results for the null hypothesis that the three international stock indices span the futures contracts for investors with either a logarithmic or power ($c = -3$) utility function, but not for investors with a mean-variance utility function. Except for corn futures the hypothesis of spanning can now be rejected for all futures contracts in our sample. The t-statistics for the $\widehat{\gamma}$'s show that neither for the logarithmic utility nor for the power utility function are the $\widehat{\gamma}$'s significantly different from zero for most futures contracts. Given the rather high correlation between the estimated power and logarithmic utility kernels (which is 0.78) this probably reflects a multicollinearity problem.

Summarizing, depending on the utility functions of interest, most of the futures contracts considered here appear to have added value for a US-investor who invests in the three stock indices.

Most commodity futures show an abnormal performance relative to mean-variance or logarithmic utility functions, whereas the currency futures only show an abnormal performance relative to power utility functions.

3.4.2 Unconditional tests in case there are nontraded assets

As outlined in Section 3.3, if an investor has a position in a nontraded asset, this will change his investment opportunity set. Therefore, in this section we will test whether the set of three international stock indices spans the futures contracts for investors with mean-variance utility functions when there are nontraded assets. For the three currency futures we will also test spanning for a power utility function ($c = -3$). We consider the case of agents that have a nonmarketable position in one of the assets underlying the futures contracts considered here. The size of the position is assumed to be 25% of the wealth invested in the three stock indices (or, equivalently, 20% of total wealth).

Table 3.3: Tests for mean-variance spanning with nontraded assets

The numbers in the table are p-values associated with the Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for an investor with a mean-variance utility function and who has a nonmarketable position in an asset. The nonmarketable position is equal to 25% of invested wealth. The column heading indicates the asset in which there is a position. Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

Panel A

	25% nontraded position:						
future:	wc	cn	sy	sm	bo	lc	lh
wheat	0.000	0.001	0.003	0.014	0.000	0.037	0.070
corn	0.021	0.000	0.005	0.018	0.001	0.571	0.514
soybeans	0.335	0.046	0.014	0.014	0.010	0.619	0.698
soyb. meal	0.961	0.592	0.379	0.129	0.705	0.726	0.628
soyb. oil	0.509	0.021	0.024	0.480	0.000	0.723	0.823
live cattle	0.009	0.033	0.023	0.026	0.028	0.000	0.000
live hogs	0.085	0.100	0.072	0.079	0.060	0.006	0.000
DMark	0.748	0.861	0.730	0.592	0.856	0.647	0.730
BPound	0.823	0.667	0.651	0.504	0.783	0.670	0.771
JYen	0.328	0.304	0.353	0.271	0.323	0.267	0.267

The p-values associated with the Wald test-statistics in Table 3.3 are for the null hypothesis that there is mean-variance spanning. The results show that whenever there is an exposure in a commodity, adding a futures contract on that same commodity almost always adds value for mean-variance investors. Of course this is what can be expected a priori. Somewhat surprisingly, the currency futures do not appear to have any abnormal performance for mean-variance investors, even though there is a 25% exposure to the foreign currency. The explanation for this result may be that the returns on the international stock indices already contain a currency component which makes mean-variance investors to choose their portfolio in such a way that adding currency futures is not useful.

Table 3.3: Tests for mean-variance spanning with nontraded assets (continued)

Panel B			
	25% nontr. position:		
future:	dm	bp	jy
wheat	0.055	0.108	0.038
corn	0.680	0.571	0.644
soybeans	0.622	0.664	0.634
soyb. meal	0.733	0.737	0.767
soyb. oil	0.835	0.923	0.779
live cattle	0.014	0.020	0.012
live hogs	0.066	0.074	0.056
DMark	0.118	0.231	0.287
BPound	0.200	0.112	0.357
JYen	0.127	0.170	0.034

Table 3.3 also shows that the five agricultural futures, wheat, corn, soybeans, soybean meal, and soybean oil, are related, in the sense that if there is an exposure in one of the five agriculturals, spanning is usually rejected for most of the agricultural futures contracts. The same is true for live cattle and live hogs. Clearly, a position in a nontraded commodity changes the investment opportunity set in such a way that the MVF of the adjusted stock indices with a futures contract added is no longer spanned by the adjusted stock indices only. Almost all investors with a mean-variance utility function can benefit from adding futures contracts to their portfolio according to their position in nontraded assets.

Though not reported here, tests whether there is spanning for investors with logarithmic utility or power utility functions show that in that case the results are very similar to the results for mean-variance spanning only. For the commodity futures the spanning hypothesis is almost always rejected whenever there is a nonmarketable position in the commodity underlying the futures contract. Also, a nonmarketable position in an agricultural commodity usually implies that the spanning hypothesis is rejected for most agricultural futures contracts, and again the same is true for live cattle and live hogs.

The main difference arises when testing spanning of currency futures by the three international stock indices in case of power utility ($c = -3$) functions. Table 3.4 shows the results for the null-hypothesis that there is both mean-variance and power utility spanning for the currency futures.

Table 3.4: Tests for mean-variance and power utility spanning with nontraded assets; currency futures only

The numbers in the table are p-values associated with the Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span three currency futures contracts for an investor with a mean-variance utility function or a power utility function with risk aversion coefficient -3, and who has a nonmarketable position in an asset. The nonmarketable position is equal to 25% of invested wealth. The first column indicates the asset in which there is a position. The initial set of assets are the S&P 500, the FTSE (UK) and the FAZ (Germany). Results are based on semi-monthly observations of monthly holding period returns from January 1984 until December 1993, resulting in a total of 240 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

	German Mark	British Pound	Japanese yen
wc	0.000	0.000	0.000
cn	0.000	0.006	0.000
sy	0.000	0.000	0.001
sm	0.002	0.005	0.014
bo	0.000	0.000	0.001
lc	0.000	0.000	0.000
lh	0.000	0.000	0.000
dm	0.000	0.000	0.001
bp	0.000	0.000	0.000
jy	0.000	0.000	0.000

The p-values in Table 3.4 show that the spanning hypothesis is now rejected for the currency futures, whatever the nonmarketable position is. Thus, unlike investors with a mean-variance or logarithmic utility function, investors with a power utility function, showing a preference for skewness, can benefit from adding currency futures to their portfolio, while we can not draw this conclusion for the other utility functions, even though there is a 25% exposure to the foreign currencies.

3.4.3 Conditional tests of spanning

As indicated in Section 3.3.3, there is ample evidence that futures returns can be predicted from the net positions of large hedgers in the futures markets, known as hedging pressure. These positions are reported by the Commodity Futures Trading Commission (CFTC). In this section we will test whether the three international stock indices span the futures contracts in our sample, if we use hedging pressure variables to predict futures returns.

In order to use this kind of conditioning information, we construct a hedging pressure variable $x_{i,t}$ for commodity or currency i as the difference between the positions of large hedgers that are short in futures contract i at time t, and the positions of large hedgers that are long in futures contract i at time t, divided by the total position of these hedgers in contract i. Thus, the hedging pressure, $x_{i,t}$, is always between -1 and +1 and represents the net position of large hedgers as a fraction of the total position of large hedgers. Using this variable, the models in equations (3.16) and (3.17) are estimated. Because data on hedging pressure are in our dataset since January 1986 only, the empirical tests that use the hedging pressure variable are for the period from January 1986 until December 1993.

Table 3.5: Tests for conditional spanning

The numbers in the table are p-values associated with the Wald test statistics for the null hypothesis that a basic set of assets consisting of the S&P 500, the FAZ (Germany), and the FTSE (UK) span futures contracts for investors with a mean-variance utility function, a mean variance and a log-utility function, a mean-variance and a power utility function with risk aversion coefficient 3, or a mean-variance, log, and power-utility function. The tests allow for return predictability based on hedging pressure as conditioning information. Results are based on semi-monthly observations of monthly holding period returns from January 1986 until December 1993, resulting in a total of 192 observations. All test statistics are based on a Newey-West covariance matrix with one lag.

Panel A

	mean-variance			**mv+log utility**		
	hedging pressure			*hedging pressure*		
	−0.50	0.00	+0.50	−0.50	0.00	+0.50
wheat	0.158	0.135	0.209	0.013	0.013	0.022
corn	0.268	0.209	0.197	0.427	0.356	0.339
soybeans	0.586	0.677	0.527	0.059	0.044	0.060
soybean meal	0.096	0.647	0.115	0.070	0.240	0.079
soybean oil	0.001	0.006	0.000	0.000	0.000	0.000
live cattle	0.017	0.047	0.126	0.040	0.099	0.228
live hogs	0.004	0.012	0.923	0.002	0.010	0.710
Deutsche Mark	0.000	0.856	0.000	0.000	0.946	0.000
British Pound	0.000	0.619	0.018	0.000	0.653	0.031
Japanese Yen	0.000	0.914	0.000	0.000	0.921	0.000

Table 3.5 shows the results of the spanning tests with conditioning information. The first three columns of Panel A show test-statistics for the hypothesis that there is mean-variance spanning for three different values of $x_{i,t}$: -0.50, 0.00, +0.50. For instance, the first column gives the p-value associated with the Wald test-statistic for the null hypothesis whether there is spanning if the hedging pressure variable is -0.50, i.e., if 75% of the positions of hedgers are long positions and 25% of the positions are short positions. The first three columns show that for three commodity futures and for the three currency futures, mean-variance spanning can be rejected convincingly, given the appropriate economic conditions. Especially for currency futures this is in sharp contrast with the results in the previous tables. Note that for currency futures mean-variance spanning is rejected when the hedging pressure variable is either +0.50 or -0.50, i.e., when hedgers are either predominantly on the long or the short side of the market, but not when the hedging pressure

Table 3.5: Tests for conditional spanning (continued)

Panel B	**mv+power utility**			**mv+log+power utility**		
	hedging pressure			*hedging pressure*		
	−0.50	0.00	+0.50	−0.50	0.00	+0.50
wheat	0.161	0.127	0.189	0.009	0.011	0.014
corn	0.355	0.290	0.355	0.407	0.352	0.339
soybeans	0.717	0.753	0.706	0.057	0.044	0.067
soybean meal	0.006	0.082	0.007	0.000	0.000	0.000
soybean oil	0.001	0.000	0.007	0.000	0.000	0.000
live cattle	0.000	0.000	0.000	0.000	0.000	0.000
live hogs	0.004	0.014	0.313	0.000	0.000	0.001
Deutsche Mark	0.000	0.000	0.000	0.000	0.000	0.000
British Pound	0.000	0.000	0.000	0.000	0.000	0.000
Japanese Yen	0.000	0.000	0.000	0.000	0.000	0.000

variable is 0.00, i.e., when the positions of hedgers are spread evenly over the long and short side of the market.

Columns 4, 5, and 6 of Table 3.5, Panel A, show similar tests for the hypothesis that there is spanning for both mean-variance and logarithmic utility investors and columns 1, 2, and 3 of Panel B show tests for the hypothesis that there is spanning for both mean-variance and power utility investors. Finally, the last three columns of Panel B show the tests for the hypothesis that there is spanning for all utility functions considered: mean-variance, logarithmic utility, and power utility. Note that because our conditional tests require that we include an intercept and x_t in the regression, our test procedure automatically tests for mean-variance spanning besides the other utility functions included. The results for these tests confirm the findings for mean-variance spanning. The major difference occurs in the first and the last three columns of Panel B, that show that when logarithmic utility functions are included spanning can also be rejected for wheat and soybean futures, which is not the case for the other utility functions.

3.5 Summary and conclusions

In this chapter it is shown how a regression framework can be used to test for mean-variance spanning as well as spanning for more general classes of utility functions. It is also shown that in a regression framework it is straightforward to test for spanning in case of zero-investment securities like futures, forwards, and swaps and in case some assets are nontraded. If zero-investment securities are considered, then spanning implies restrictions on the coefficients in the spanning regression that reflect the zero-investment property. If an investor has a position in a nontraded asset, then this changes his investment opportunity set. In spanning tests this can be incorporated by using returns that are adjusted for the return on the nontraded asset. One of the advantages of a regression framework is that the spanning regression provides clear interpretations in terms of performance measurement and optimal portfolio choice. We test whether three international stock indices, i.e., the S&P 500, the FAZ (Germany), and the FTSE (UK), span a set of commodity futures and currency futures. If it is assumed that all relevant moments of monthly holding returns are constant, and that there are no market frictions like short selling constraints and transaction costs, then we can reject the hypothesis that there is spanning for most futures contracts, but whether or not the spanning hypothesis is rejected depends on the specific utility functions of interest. If an investor has a nonmarketable position in a commodity underlying one of the futures contracts, then spanning can almost always be rejected for the futures contract on that same commodity for all utility functions considered. Moreover, a nonmarketable position in one agricultural commodity usually implies that the hypothesis of spanning is rejected for most of the agricultural futures contracts. If there is an exposure to a foreign currency, then spanning can only be rejected for investors with power utility functions that reflect a preference for skewness. Finally, allowing expected returns to depend on the net positions of large hedgers in the futures market, spanning can be rejected for many futures contracts for all utility functions considered.

Appendix 3.A Proof of Proposition 1

In this appendix we present the proof of Proposition 1 allowing for zero-investment securities. Let $w \in \widetilde{W}$, where $\widetilde{W} = W_e^S$ if r_{t+1} refers to a non-zero investment security and $\widetilde{W} = W_e^F$ if r_{t+1} refers to a zero-investment security. Also let e_r be 1 if r_{t+1} refers to a non-zero investment security and 0 if r_{t+1} refers to a zero-investment security. Denote the minimum second moment kernel as $M_{t+1}^0 = R'_{t+1}E[R_{t+1}R'_{t+1}]^{-1}e_K$ and denote the projection of r_{t+1}on R_{t+1} and all $M_{t+1} \in \mathcal{M}$ as $\widehat{r}_{t+1}$:

$$\widehat{r}_{t+1} = w'R_{t+1} + \gamma M_{t+1}.$$

There is $\mathcal{M}$-spanning if and only if $w \in \widetilde{W}$ (i.e., $w'e_K = e_r$) and if $\gamma = 0$. Notice that the least squares projection implies that

$$E[(r_{t+1} - w'R_{t+1} - \gamma M_{t+1})R'_{t+1}] = 0, \tag{A.1a}$$

$$E[(r_{t+1} - w'R_{t+1} - \gamma M_{t+1})M_{t+1}] = 0. \tag{A.1b}$$

Also note that since $M_{t+1}^0 \in \mathcal{M}$, we have that

$$E[(r_{t+1} - w'R_{t+1} - \gamma M_{t+1})M_{t+1}^0] = 0. \tag{A.2}$$

Proof. First (*sufficiency*), assume that $w'e_K = e_r$ and that $\gamma = 0$. Then it follows from (A.1b) and (A.2) that

$$E[r_{t+1}M_{t+1}] - w'e_K - 0 = 0 \Leftrightarrow E[r_{t+1}M_{t+1}] = e_r.$$

which shows that there is $\mathcal{M}$-spanning.

Next (*necessity*), suppose that there is $\mathcal{M}$-spanning, i.e., that $E[r_{t+1}M_{t+1}] = e_r, \forall M_{t+1} \in \mathcal{M}$. From (A.1b) it then follows that for each $M_{t+1} \in \mathcal{M}$:

$$\begin{aligned} 0 &= E[r_{t+1}M_{t+1}] - w'E[R_{t+1}M_{t+1}] - \gamma E[M_{t+1}^2] \Longleftrightarrow \\ e_r - w'e_K &= \gamma E[M_{t+1}^2]. \end{aligned}$$

Similarly, from (A.2) it follows that

$$\begin{aligned} 0 &= E[r_{t+1}M_{t+1}^0] - w'E[R_{t+1}M_{t+1}^0] - \gamma E[M_{t+1}M_{t+1}^0] \Longleftrightarrow \\ e_r - w'e_K &= \gamma E[M_{t+1}R'_{t+1}E[R_{t+1}R'_{t+1}]^{-1}e_K] = \gamma e'_K E[R_{t+1}R'_{t+1}]^{-1}e_K. \end{aligned}$$

Combining the latter two equations gives that $\gamma = 0$, implying that $w'e_K = e_r$, which completes the proof.

Appendix 3.B Derivation of the limit distribution of the OLS-estimates in (3.8)

In this appendix we will derive (under sufficient regularity conditions) the limit distribution for the OLS-estimates of the regression parameters in (B.1). Recall that we can test for spanning for a certain utility function U by testing whether in the regression

$$r_{t+1} = \alpha' R_{t+1} + \gamma U'(R'_{t+1}\varphi) + \varepsilon_{t+1}, \tag{B.1}$$

$\alpha'\iota = 1$ and $\gamma = 0$. For simplicity, we consider the case where there is only one utility function and we have reparametrized $cU'(w^{*\prime} R_{t+1})$ as $U'(\varphi' R_{t+1})$. In empirical applications the parameters φ have to be estimated. From

$$E_t[R_{t+1}U'(R'_{t+1}\varphi)] = \iota,$$

estimates of φ can be obtained with GMM using the sample moments:

$$g(\varphi, R) \equiv \frac{1}{T}\sum_{t=1}^{T} g_t(\varphi, R_t) = \frac{1}{T}\sum_{t=1}^{T} U'(R'_t\varphi)R_t - \iota, \tag{B.2}$$

where $g_t(\varphi, R_t) = U'(R'_t\varphi)R_t - \iota$. Notice that this system is exactly identified since there are K portfolio weights w and K first order conditions. Denote

$$G(\varphi, R) = \partial g(\varphi, R)/\partial\varphi = \frac{1}{T}\sum_{t=1}^{T} U''(R'_t\varphi)R_t R'_t.$$

Then we have that

$$\sqrt{T}(\widehat{\varphi} - \varphi) \approx -G(\varphi, R)^{-1}\sqrt{T}g(\widehat{\varphi}, R). \tag{B.3}$$

Denoting the limiting covariance matrix of $\sqrt{T}g(\varphi, R)$ as S_{gg}, and the probability limit of $G(\varphi, R)$ as A, then the limit distribution of $\widehat{\varphi}$ is given by:

$$\sqrt{T}(\widehat{\varphi} - \varphi) \xrightarrow{L} N(0, A^{-1}S_{gg}A^{-1\prime}).$$

As a next step, rewrite (B.1) as

$$\begin{aligned} r_{t+1} &= \alpha' R_{t+1} + \gamma U'(R'_{t+1}\widehat{\varphi}) + \varepsilon_{t+1} + \gamma\{U'(R'_{t+1}\varphi) - U'(R'_{t+1}\widehat{\varphi})\} \\ &= \alpha' R_{t+1} + \gamma U'(R'_{t+1}\widehat{\varphi}) + \varepsilon_{t+1} + u_{t+1}, \end{aligned}$$

which defines u_{t+1}. Defining $x_t \equiv (R_t', \; U\text{'}(R_t'\varphi))'$ and $\widehat{x}_t \equiv (R_t', \; U\text{'}(R_t'\widehat{\varphi}))'$, the estimates $\widehat{\alpha}$ and $\widehat{\gamma}$ satisfy:

$$\sqrt{T}\begin{pmatrix} \widehat{\alpha}-\alpha \\ \widehat{\gamma}-\gamma \end{pmatrix} = \left[\frac{1}{T}\sum_{t=1}^{T}\widehat{x}_t\widehat{x}_t'\right]^{-1}\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\widehat{x}_t\left(\varepsilon_t+u_t\right)\right). \tag{B.4}$$

Since under the null hypothesis that there is spanning $\gamma = 0$, the error term u_t equals 0, and hence does not affect the limit distribution. Using a linear expansion (as in Pagan (1984)) we obtain for the last factor in (B.4) that

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\widehat{x}_t\varepsilon_t \approx \frac{1}{\sqrt{T}}\sum_{t=1}^{T}x_t\varepsilon_t + \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix} 0 \\ U\text{"}(R_t'\varphi)R_t'\varepsilon_t \end{pmatrix}\sqrt{T}(\widehat{\varphi}-\varphi). \tag{B.5}$$

Substituting (B.3) into (B.5) gives:

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\widehat{x}_t\varepsilon_t \approx \frac{1}{\sqrt{T}}\sum_{t=1}^{T}x_t\varepsilon_t - \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix} 0 \\ U_t\text{"}(R_t'\varepsilon_t) \end{pmatrix}G(\varphi,R)^{-1}\sqrt{T}g(\varphi,R).$$

If we denote the limit distribution of this term with $N(0,V)$, then the limit distribution of the regression parameters is given by:

$$\sqrt{T}\begin{pmatrix} \widehat{\alpha}-\alpha \\ \widehat{\gamma}-\gamma \end{pmatrix} \xrightarrow{L} N(0,Q),$$

where an estimate of Q can be obtained from

$$\widehat{Q} = \left[\frac{1}{T}\sum_{t=1}^{T}\widehat{x}_t\widehat{x}_t'\right]^{-1}\widehat{V}\left[\frac{1}{T}\sum_{t=1}^{T}\widehat{x}_t\widehat{x}_t'\right]^{-1}.$$

In our applications we estimated V by $\widehat{V}$ following Newey & West (1987a), using Bartlett weights in estimator.

Appendix 3.C Interpretation of the slope coefficients in the spanning regression

In this appendix we will show how the interpretation of the slope coefficients γ in (3.8) can be derived. For simplicity and without loss of generality, we consider the case where M_{t+1} is a scalar, i.e., where the interest is in one non mean-variance utility function only:

$$r_{t+1} = \alpha + \beta' R_{t+1} + \gamma M_{t+1} + \varepsilon_{t+1}. \tag{C.1}$$

Denote with $\widehat{M}_{t+1}$ the projection of M_{t+1} on the asset returns R_{t+1} and a constant, i.e., $\widehat{M}_{t+1} =$ Proj$[M_{t+1} \mid 1, R_{t+1}]$. Thus, $\widehat{M}_{t+1}$is the minimum variance kernel that has the same expectation as M_{t+1}, $v = E[M_{t+1}]$. Using partitioned inverses it is straightforward to show that the parameter γ in (C.1) can be written as

$$\gamma = \frac{Cov[M_{t+1} - \widehat{M}_{t+1}, r_{t+1}]}{Var[M_{t+1} - \widehat{M}_{t+1}]}. \tag{C.2}$$

Next notice that since the minimum variance kernel $\widehat{M}_{t+1}$ is proportional to the marginal (derived) utility of a mean-variance investor whose zero-beta rate equals $1/v$, the generalized Jensen measure $\alpha_J(v)$ of r_{t+1} relative to the benchmark assets R_{t+1} can be derived from

$$v\alpha_J(v) = E[\widehat{M}_{t+1} r_{t+1}] - 1.$$

It is now straightforward to obtain for the performance measure $\lambda_r = E[r_{t+1} M_{t+1}] - 1$:

$$\begin{aligned} \lambda_r &= E[r_{t+1} M_{t+1}] - 1 \\ &= E[r_{t+1} \widehat{M}_{t+1}] - 1 + E[r_{t+1}(M_{t+1} - \widehat{M}_{t+1})] \\ &= v\alpha_J(v) + \gamma Var[M_{t+1} - \widehat{M}_{t+1}], \end{aligned}$$

which completes the derivation.

Chapter 4

Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets

4.1 Introduction

A crucial assumption in the spanning tests that were discussed in the previous chapters, as well as almost all tests for extension of the efficient set that have been proposed in the literature, is the absence of market frictions such as short sales restrictions and transaction costs. For many investors, however, such frictions are important facts of life. The aim of this chapter is to extend the tests for mean-variance spanning and intersection in order to take these market frictions into account. The chapter is therefore related to Hansen, Heaton & Luttmer (1995) who derive the asymptotic distribution of specification error bounds allowing for market frictions, and to Luttmer (1996) who analyzes the impact of market frictions on volatility bounds. Especially the region subset test considered by Hansen, Heaton & Luttmer is closely related to some results presented in this chapter. They do not consider testing for spanning however. Glen & Jorion (1993) have proposed an alternative way to test for spanning in case of short sales constraints on the additional assets, but their test is more restrictive than ours in a number of ways. A detailed comparison of these test procedures and the one proposed in this chapter will be presented in Section 4.3. Transaction costs and short sales constraints are important in many investment problems, but perhaps their presence is most predominant in the case of emerging markets. Using the Emerging Market Data Base (EMDB) of the International Finance Corporation (IFC) both DeSantis (1994) and Harvey (1995) show that the mean-variance frontier that is based on well-developed western markets only, significantly shifts outward when the emerging markets are included. However, these results presuppose that there are no transaction costs or any other market frictions for both the developed and the emerging markets. Using returns on closed-end country funds Bekaert & Urias (1996) try to overcome this problem, since the returns on these funds are attainable to investors. Based on emerging market country funds Bekaert & Urias find only mixed evidence for the diversification benefits of emerging markets. Although the use of country funds adjusts for the effect of transaction costs and short sales constraints that investors face in emerging markets, it does not account for short sales constraints and transaction costs on the country funds themselves or on the benchmark assets.

In this chapter, we provide direct evidence on the effect of transaction costs and short selling constraints on the diversification benefits of emerging markets, by using the same IFC Indices as in DeSantis (1994) and Harvey (1995), but incorporating these market frictions in our testing methodology. Our results show that the test statistics are affected in a nontrivial way by the presence of short sales constraints and transaction costs and that it is important to account for these effects in both the emerging markets as well as the benchmark assets. Although the evidence against mean-variance spanning is weaker when short sales constraints on both the emerging markets and the benchmark assets are taken into account, the hypothesis of spanning can still be rejected for many emerging markets. However, when incorporating transaction costs it is much harder to reject the hypothesis of spanning, at least when investors trade their portfolio on a monthly basis. For investors that trade their portfolio less frequently their is still evidence in favor of the diversification benefits of emerging markets. For investors that have an investment horizon of one month, the critical level of transaction costs above which the hypothesis of spanning can not be rejected is usually smaller than the estimates of the size of these transaction costs that have been reported in the literature. Even though the hypothesis of spanning is still rejected for a number of emerging markets when there are short sales constraints and transaction costs, which suggests that diversification benefits are still possible, these results must be interpreted with caution since foreign ownership restrictions may prevent investors from realizing these benefits. Indeed, when performing some of the spanning tests for the IFC Investable Indices, which take foreign ownership restrictions into account but which are available for a shorter sample period only, there is hardly any evidence left against the hypothesis of spanning.

The plan of this chapter is as follows. In Section 4.2 we first of all formulate the hypotheses of mean-variance spanning and intersection in case of short sales restrictions. Regression-based tests for these hypotheses are proposed in Section 4.3. In Section 4.4 the analysis is extended to the case of transaction costs. The empirical results on investing in emerging markets are presented in Section 4.5 and in the final section we will offer some concluding remarks.

4.2 Mean-variance spanning with short sales constraints

As in the previous chapters, consider a set of K assets, whose gross returns are given by the vector R_{t+1}. Investors can hold portfolios $w \in C \subset I\!R^K$ such that $w'\iota_K = 1$, where ι_K is a K-vector containing only ones. The set of returns available to investors is therefore given by:

$$X = \{R^p_{t+1} : R^p_{t+1} = w'R_{t+1},\ w \in C,\ \text{and}\ w'\iota_K = 1\}.$$

Let us first of all reconsider the case where there are no market frictions, i.e., $C = \mathbb{R}^K$. In this case we can obtain the results in the previous chapters from (2.1):

$$E[M_{t+1}R_{t+1} \mid I_t] = \iota_K.$$

In this chapter we will restrict ourselves to unconditional versions of (2.1) and to unconditional mean-variance spanning Now consider the presence of market frictions such as short sales constraints and transaction costs. These can be dealt with by letting C be a particular subset of $\mathbb{R}^K$ and/or by adjusting the vector of returns R_{t+1} to reflect the frictions. In case of short sales constraints for instance, $C = \mathbb{R}^K_+$, the nonnegative part of $\mathbb{R}^K$. When there are short sales constraints on the portfolio holdings, the condition in (2.1) must be replaced by:

$$E[m(v)_{t+1}R_{t+1}] \leq \iota_K. \tag{4.1}$$

The mean-variance efficient frontier without short sales can be found by solving the problem:

$$\begin{aligned} \max_{\{w\}} w'E[R_{t+1}] - \frac{1}{2}\gamma w'Var[R_{t+1}]w&, \qquad (4.2)\\ \text{s.t. } w'\iota_K \;=\; 1 \text{ and } w_i \geq 0, \forall i&. \end{aligned}$$

From the Kuhn-Tucker conditions, mean-variance efficient portfolios w^* satisfy:

$$\begin{aligned} E[R_{t+1}] - \eta\iota_K + \delta &= \gamma Var[R_{t+1}]w^*, \qquad (4.3)\\ \delta_i &= 0 \text{ if } w_i > 0,\\ \delta_i &\geq 0,\\ \delta_i w_i &= 0,\ \forall i. \end{aligned}$$

The vector δ contains the Kuhn-Tucker multipliers for the restrictions that the portfolio weights are nonnegative. The Lagrange multiplier for the restriction that $w'\iota_K = 1$, is equal to η, the intercept of the line that is tangent to the mean-variance frontier in mean-standard deviation space.

Now take the mean-variance efficient portfolio for which $\eta = 1/v$, with v the expectation of a stochastic discount factor that prices R_{t+1} correctly subject to short sales constraints. Denote by $R^{(v)}_{t+1}$ the L-dimensional subvector of R_{t+1} that only contains the returns of the assets for which the short sales constraints in (4.3) are not binding. It is straightforward to show that the mean-variance efficient portfolio in (4.3) is equal to the mean-variance efficient portfolio without short sales constraints of the assets in $R^{(v)}_{t+1}$ only:

$$\begin{aligned} E[R^{(v)}_{t+1}] - \frac{1}{v}\iota_L &= \gamma^{(v)}Var[R^{(v)}_{t+1}]w^{(v)} \text{ and} \qquad (4.4)\\ E[R_{t+1}] - \frac{1}{v}\iota_K + \delta^{(v)} &= \gamma^{(v)}Cov[R_{t+1}, R^{(v)}_{t+1}]w^{(v)}, \end{aligned}$$

where $Cov[R_{t+1}, R_{t+1}^{(v)}]$ is de $K \times L$-dimensional covariance matrix of R_{t+1} and its subvector $R_{t+1}^{(v)}$. Thus the mean-variance efficient portfolio for a set of assets with return vector R_{t+1} subject to short sales constraints, is simply the mean-variance efficient portfolio for the subset of assets for which the restrictions are not binding (see, e.g., Markowitz (1991)). Observe that for the assets that are in $w^{(v)}$ the Kuhn-Tucker multipliers $\delta_i^{(v)}$ in (4.4) are zero.[13]

Since the mean-variance stochastic discount factor is a linear function of the mean-variance efficient portfolio, in case of short sales restrictions the mean-variance stochastic discount factor that prices R_{t+1}, $m_R(v)_{t+1}$, is equal to

$$\begin{aligned} m_R(v)_{t+1} &= v + \alpha^{(v)\prime}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]), \\ \alpha^{(v)} &= Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]). \end{aligned} \tag{4.5}$$

The L-dimensional vector of projection coefficients $\alpha^{(v)}$ is of course proportional to the vector of mean-variance efficient portfolio weights $w^{(v)}$: $w^{(v)} = \alpha^{(v)}/\iota_L'\alpha^{(v)} = -\alpha^{(v)}/\gamma^{(v)}v$. It is shown in Appendix 4.A that the stochastic discount factor as defined in (4.5) has the lowest variance of all stochastic discount factors that have expectation v and that price R_{t+1} correctly subject to short sales constraints, as long as $v > 0$. Therefore, in case of short sales constraints the duality between mean-variance frontiers and volatility bounds still holds.

Next consider a set of N additional assets with return vector r_{t+1} besides the set of K benchmark assets with return vector R_{t+1}. From Chapter 2, mean-variance spanning of the assets r_{t+1} by the benchmark assets R_{t+1} occurs if the mean-variance stochastic discount factors that price R_{t+1} correctly, also price r_{t+1}, i.e., if:

$$E[m_R(v)_{t+1} r_{t+1}] \le \iota_N, \tag{4.6}$$

holds for all values of v. Substituting (4.5) into (4.6), this is equivalent to:

$$vE[r_{t+1}] + Cov[r_{t+1}, R_{t+1}^{(v)}]Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]) \le \iota_N. \tag{4.7}$$

The inequality sign in (4.6) reflects the fact that there are short sales constraints on r_{t+1}. In the absence of short sales constraints on r_{t+1}, the inequality becomes an equality. If there is only one value of v for which (4.6) holds, then there is intersection. If $m_R(v)_{t+1}$ prices r_{t+1}, agents whose marginal utility corresponds with $m_R(v)_{t+1}$, can not increase their utility by including the assets r_{t+1} in their portfolio besides the benchmark assets R_{t+1}. Because of the short sales constraints

[13] It can easily be seen from (4.4) that if we take the portfolio $w^{(v)}$ as the benchmark portfolio, the vector of Kuhn-Tucker multipliers $\delta^{(v)}$ is proportional to the vector $\alpha_J(v)$ of Jensen's alphas of the returns R_{t+1} with respect to this benchmark portfolio. Recall that the vector of Jensen's alphas can be obtained as the intercept in a regression of the excess returns $R_{t+1} - \frac{1}{v}\iota_K$ on the excess returns $R_{t+1}^{(v)} - \frac{1}{v}\iota_L$ and a constant.

agents can only increase their utility by including an asset with return $r_{i,t+1}$ if $E[m_R(v)_{t+1}r_{i,t+1}] > 1$.

4.3 Testing for intersection and spanning

4.3.1 Testing for intersection

Absent short sales constraints and any other market frictions, the hypotheses of mean-variance intersection and spanning are equivalent to the condition that

$$E[m_R(v)_{t+1}r_{t+1}] = \iota_N, \tag{4.8}$$

for one value of v (intersection) or for all values of v (spanning), where

$$m_R(v)_{t+1} = v + (\iota_K - vE[R_{t+1}])'Var[R_{t+1}]^{-1}(R_{t+1} - E[R_{t+1}]).$$

Recall from the previous chapters that in this case tests for intersection and spanning can be based on the regression

$$r_{t+1} = a + BR_{t+1} + \varepsilon_{t+1}, \tag{4.9}$$

with $E[\varepsilon_{t+1}] = 0$ and $E[\varepsilon_{t+1}R_{t+1}] = 0$. Intersection for a given value of v implies that $av + (B\iota_K - \iota_N) = 0$, while spanning implies that $a = 0$ and $B\iota_K - \iota_N = 0$.

As shown in the previous section, if there are short sales restrictions on the benchmark assets R_{t+1}, the stochastic discount factor $m_R(v)_{t+1}$ is a linear function of $R_{t+1}^{(v)}$ only, and if there are short sales restrictions on the additional assets r_{t+1}, then the equality in (4.8) becomes an inequality. For a given value of v, the restrictions implied by intersection can be derived by substituting (4.5) into (4.6), which results in (4.7). These restrictions are equivalent to the restrictions that in the regression

$$r_{t+1} = a^{(v)} + B^{(v)}R_{t+1}^{(v)} + \varepsilon_{t+1}^{(v)}, \tag{4.10}$$

it holds true that

$$va^{(v)} + (B^{(v)}\iota_L - \iota_N) \leq 0. \tag{4.11}$$

Intuitively, since in case of short sales constraints the mean-variance efficient portfolio of R_{t+1} for a given value of v consists of positions in only those assets for which the constraints are not binding, intersection requires that there is intersection at the unrestricted frontier of $R_{t+1}^{(v)}$ rather than at the unrestricted frontier of R_{t+1}. The inequality in (4.11) reflects the short sales constraints on r_{t+1}. If some elements on the left hand side of (4.11) are negative this would imply that a more efficient portfolio could be reached by taking short positions in the corresponding elements of r_{t+1}. Since such portfolios are unattainable with short sales constraints however, the inequality sign reflects that this situation would not violate the hypothesis that there is intersection.

A Wald test can be used to test the inequality constraints in (4.11) (see, e.g., Kodde & Palm (1986)). Denote the left hand side of (4.11) as $v\alpha_J(v)$, where $\alpha_J(v)$ is the N-dimensional vector of Jensen's alphas of the assets r_{t+1} relative to the mean-variance efficient portfolio of $R_{t+1}^{(v)}$ with zero-beta return $1/v$. The sample equivalent of $\alpha_J(v)$ is $\widehat{\alpha}_J(v)$, and the $N \times N$ covariance matrix of $v\widehat{\alpha}_J(v)$, $Var[v\widehat{\alpha}_J(v)]$, can be obtained from the restricted covariance matrix of the OLS-estimates of (4.10), where the restrictions are given by $va^{(v)} + (B^{(v)}\iota_L - \iota_N) = 0$. Following Kodde & Palm, under the null hypothesis and standard regularity conditions, the test statistic

$$\xi(v) = \min_{\{\alpha_J(v)\leq 0\}} (\widehat{\alpha}_J(v) - \alpha_J(v))' Var[\widehat{\alpha}_J(v)]^{-1} (\widehat{\alpha}_J(v) - \alpha_J(v)), \tag{4.12}$$

is asymptotically distributed as a mixture of χ^2 distributions. For the case considered here, where we test whether there is intersection for the N assets r_{t+1}, the probability of $\xi(v)$ exceeding a given value c is, under the null-hypothesis, given by (see, e.g., Kodde & Palm (1986)):

$$\Pr\{\xi(v) \geq c\} = \sum_{i=0}^{N} \Pr\{\chi_i^2 \geq c\} w(N, i, Var[\widehat{\alpha}_J(v)]), \tag{4.13}$$

where $w(N, i, Var[\widehat{\alpha}_J(v)])$ are probability weights[14]. Given an estimate of the covariance matrix $Var[\widehat{\alpha}_J(v)]$, the probabilities can be determined using numerical simulation, as proposed by Gouriéroux et al. (1982). Alternatively, without calculating the weights, Kodde & Palm (1986) show that an upper and a lower bound on the p-values of $\xi(v)$ are given by

$$\begin{aligned} p_{up}[\xi(v)] &= \frac{1}{2}\Pr\{\chi_{N-1}^2 \geq \xi(v)\} + \frac{1}{2}\Pr\{\chi_N^2 \geq \xi(v)\} \\ p_{low}[\xi(v)] &= \frac{1}{2}\Pr\{\chi_1^2 \geq \xi(v)\}. \end{aligned} \tag{4.14}$$

Of course, when implementing the intersection test in empirical applications, it is usually the case that for a particular value of v we do not observe which assets are in $R_{t+1}^{(v)}$, but have to derive this information from the asset returns in our sample. It is shown in Appendix 4.B that this does not affect the limit distribution of the Wald test-statistic for the restrictions in (4.11) however, if v corresponds to an efficient portfolio where none of the weights in $w^{(v)}$ is exactly zero (i.e., $w_i^* = 0$ and $\delta_i > 0$). If this latter situation occurs, then it is easily verified that the size of the test (conditional on $R_{t+1}^{(v)}$) does not depend on $R_{t+1}^{(v)}$, and hence the unconditional size equals the one

[14] The weights $w(N, i, Var[\widehat{\alpha}_J(v)])$ are the probabilities that $(N-i)$ of the N elements of a vector with a $N(0, Var[\widehat{\alpha}_J(v)])$ distribution are strictly negative.

chosen, which shows the validity of our test. Further discussion of this point will follow at the end of this section.

As in Gibbons, Ross & Shanken (1989) it can be shown that the test statistic in (4.12) also has an interpretation in terms of Sharpe ratios:

$$\xi(v) = T\frac{\widetilde{\theta}(v)^2 - \theta(v)^2}{1+\theta(v)^2}, \tag{4.15}$$

where $\theta(v)^2$ is the maximum Sharpe ratio that can be obtained from the excess returns $R^{(v)}_{t+1} - 1/v$, and $\widetilde{\theta}(v)^2$ is the maximum Sharpe ratio that can be obtained from the excess returns $R^{(v)}_{t+1} - 1/v$ and $r_{t+1} - 1/v$, with short sales constraints on r_{t+1} only. Therefore, the familiar interpretation of intersection tests in terms of performance measures as discussed in Section 5 of Chapter 2, also holds when there are short sales constraints.

At this point it is useful to compare our test procedure with the one proposed by Glen & Jorion (1993). Glen & Jorion calculate the mean-variance efficient portfolio of the benchmark assets subject to short sales constraints, with $1/v$ equal to the observed risk free rate. Because of the existence of a risk free asset the hypotheses of intersection and spanning coincide in this case. The mean returns of all assets, i.e., both R_{t+1} and r_{t+1}, are then adjusted such that the calculated portfolio is mean-variance efficient without short sales constraints. Thus, the mean returns are adjusted such that the calculated portfolios would yield Jensen's alpha's equal to zero. Using these adjusted returns and assuming normality, a new set of T returns is simulated and a test statistic based on Sharpe ratios is calculated, but with short sales constraints on all the available assets rather than on r_{t+1} only. By repeating this process many times an empirical distribution of the test statistic can be obtained. Our procedure has the advantage that it yields a known distribution for the test-statistic in (4.12). Apart from this, our procedure has the advantage that we avoid the assumption that one of the assets is riskless and that the test can also be used to test for spanning.

4.3.2 Testing for spanning

Up to now we considered tests for intersection. Spanning implies that the restrictions in (4.11) hold for all relevant values of v. Notice that for a given set of K asset returns R_{t+1}, there is only a finite number of subsets with $L^{(v)}$ elements, $L^{(v)} = 1, 2, .., K$, with $R^{(v)}_{t+1}$ the $L^{(v)}$-dimensional vector containing the returns on the subset of the assets . Let $V^{[j]}$ be the set of those values of v for which the subset of assets for which the short sales constraints in the mean-variance efficient portfolios are not binding is the same, and denote the $L^{[j]}$-dimensional vector of returns for these assets as $R^{[j]}_{t+1}$, i.e., $R^{[j]}_{t+1} = R^{(v)}_{t+1}$ if and only if $v \in V^{[j]}$. Since for $v \in V^{[j]}$ the mean-variance efficient frontier of R_{t+1} coincides with the mean-variance frontier of $R^{[j]}_{t+1}$, the mean-variance frontier of R_{t+1} with short sales constraints consists of a finite number of parts of the unrestricted mean-variance frontiers of the subsets $R_{t+1}^{[j]}$. It follows that the return on the additional assets r_{t+1} are spanned by the returns on the benchmark assets R_{t+1} if

$$E[m_R^{[j]}(v)_{t+1}r_{t+1}] \leq \iota_N, \ \ \forall j, \tag{4.16}$$

where $m_R^{[j]}(v)_{t+1}$ is the mean-variance pricing kernel that is linear in $R_{t+1}^{[j]}$. If there are only short sales constraints on the benchmark assets R_{t+1} and not on the additional assets r_{t+1}, the inequality in (4.16) becomes an equality. If there are only short sales constraints on r_{t+1} and not on R_{t+1}, $R_{t+1}^{[j]} = R_{t+1}$.

Intuitively, since if there are short sales constraints the mean-variance frontier of R_{t+1} consists of parts of the unrestricted mean-variance frontiers of the subsets of returns $R_{t+1}^{[j]}$, $j = 1, 2, ..M$, r_{t+1} can only be spanned by the returns R_{t+1} if it is spanned by the M subsets of R_{t+1}. It follows then that if there are no short sales constraints on the assets r_{t+1}, there is mean-variance spanning if and only if in the M regressions

$$r_{t+1} = a^{[j]} + B^{[j]}R_{t+1}^{[j]} + \varepsilon_{t+1}^{[j]}, \tag{4.17}$$

it holds that

$$a^{[j]} = 0 \text{ and } B^{[j]}\iota^{[j]} = \iota_N, \tag{4.18}$$

where $\iota^{[j]}$ is an $L^{[j]}$-dimensional vector consisting of ones. The hypothesis that there is spanning can therefore easily be tested by using a multivariate regression of r_{t+1} on all $R_{t+1}^{[j]}$ and using a Wald test for the Huberman-Kandel restrictions in each of these regressions. If there are also short sales restrictions on r_{t+1}, then the conditions in (4.16) imply that we should again use the multivariate regression in (4.17), but now the restrictions imposed are that

$$a^{[j]}v + B^{[j]}\iota^{[j]} \leq \iota_N, \ \ \text{for all } v \in V^{[j]}. \tag{4.19}$$

Denoting $v_{\min}^{[j]} = \min_{\{v \in V^{[j]}\}} v$, and $v_{\max}^{[j]} = \max_{\{v \in V^{[j]}\}} v$, the restrictions in (4.19) are satisfied if there is intersection for $v_{\min}^{[j]}$ and for $v_{\max}^{[j]}$, since in that case there is also intersection for all the intermediate values of $v^{[j]}$. Therefore, testing for spanning comes down to jointly testing the restrictions:

$$\begin{aligned} a^{[j]}v_{\min}^{[j]} + B^{[j]}\iota^{[j]} &\leq \iota_N, \\ a^{[j]}v_{\max}^{[j]} + B^{[j]}\iota^{[j]} &\leq \iota_N, \end{aligned} \tag{4.20}$$

for $j = 1, ..., M$. Again, the test-statistic for the inequality restrictions in (4.20) is standard and is now based on testing simultaneously for intersection for $v_{\min}^{[j]}$ and $v_{\max}^{[j]}$, $j = 1, 2, .., M$, analogous to (4.12). The p-values can be obtained from (4.13) by replacing χ^2_N with χ^2_{2MN}. Similarly, without calculating the weights in (4.13), upper and lower bounds on the p-values can be obtained from (4.14) by replacing χ^2_N and χ^2_{N-1} by χ^2_{2MN} with χ^2_{2MN-1} respectively.

The intersection and spanning tests presented here are closely related to the region subset tests in Hansen, Heaton & Luttmer (HHL) (1995). In the region subset tests of HHL the interest is in testing whether, given the initial asset returns R_{t+1}, including the additional returns r_{t+1} causes a significant shift in the volatility bound. For a given mean v of the stochastic discount factor this simply amounts to a test for intersection. The region subset test of HHL is based on the minimum variance stochastic discount factor $m(v)_{t+1}$ that prices the assets in R_{t+1} and r_{t+1} subject to short sales constraints:

$$m(v)_{t+1} = v + \alpha_R^{(v)\prime}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]) + \alpha_r^{(v)\prime}(r_{t+1}^{(v)} - E[r_{t+1}^{(v)}]).$$

This is similar to the minimum variance stochastic discount factor in (4.5), but now based on all the assets rather than the benchmark assets R_{t+1} only. The intersection hypothesis is now equivalent to the hypothesis that the coefficients associated with r_{t+1}, $\alpha_r^{(v)}$, are equal to zero. As pointed out by HHL (1995), the asymptotic distribution of the estimate of $\alpha_i^{(v)}$ is nonstandard if $\alpha_i^{(v)}$ equals zero, because in that case it is impossible to distinguish between assets that have a zero coefficient and assets whose short sales constraints are binding. Since in the region subset tests the null hypothesis is that the coefficients $\alpha_r^{(v)}$ are zero, it is under the hypothesis of interest that the limiting distribution of $\alpha_r^{(v)}$ is nonstandard (see HHL (1995) for further details).

The interest in this chapter is in the hypothesis of spanning rather than intersection. As shown above, testing for spanning with short sales constraints amounts to simultaneously testing for intersection at those values of v for which one of the weights in the mean-variance efficient portfolio of the benchmark assets $R_{t+1}^{[j]}$ is zero, i.e., in $v_{\min}^{[j]}$ and $v_{\max}^{[j]}$, which suggests that the limiting distribution for the spanning test may be nonstandard as well. Recall however, that the spanning test is based on testing for intersection in $v_{\min}^{[j]}$ and $v_{\max}^{[j]}$ because these are the two extreme values of $v^{[j]}$ for which the short sales constraints on $R_{t+1}^{[j]}$ are not binding. Intersection at $v_{\min}^{[j]}$ and $v_{\max}^{[j]}$ implies intersection at all the intermediate values of $v^{[j]}$ and therefore spanning. Thus, since the spanning test described above is essentially based on testing intersection for all values of $v^{[j]}$ for which the short sales constraints on $R_{t+1}^{[j]}$ are not binding, the problem encountered in the region subset test of HHL does not occur.

Another way to look at this is the following. HHL estimate the minimum variance stochastic discount factor $m(v)_{t+1}$ under non-negativity constraints (which essentially induces the nonstandard limit distribution) and end up with testing equality restrictions. On the other hand, our regression-based estimator is unrestricted with a standard asymptotic distribution, but we end up with - more difficult - inequality restrictions that have to be tested. This latter problem is well-studied in the literature however (see, e.g., Gouriéroux et al. (1982) and Kodde & Palm (1986)).

4.4 Mean-variance spanning with transaction costs

When taking transaction costs into account it is useful to differentiate between the return on a long position in asset i, $\tau_i^\ell R_{i,t+1}$, and the return on a short position in asset i, $\tau_i^s R_{i,t+1}$ (see, e.g., Luttmer (1996)). Let $\widetilde{R}_{t+1}$ be a $2K$-dimensional vector, the first K elements of which are the returns on the long positions in the assets $i = 1, ..., K$, and the last K elements of which are the returns on the short positions in these same assets. Thus, $\widetilde{R}_{i,t+1} = \tau_i^\ell R_{i,t+1}$ and $\widetilde{R}_{i+K,t+1} = \tau_i^s R_{i,t+1}$. One way to motivate this kind of transaction costs is to assume that investors have to pay a bid/ask spread when buying or (short) selling the asset at time t. Thus, letting $a_i > 0$ and $b_i > 0$ be the ask and the bid spread respectively, as a percentage of the price $P_{i,t}$, τ_i^ℓ is defined by $\tau_i^\ell R_{i,t+1} = P_{i,t+1}/((1 + a_i)P_{i,t})$, implying that $\tau_i^\ell = 1/(1 + a_i)$, and τ_i^s is defined by $\tau_i^s R_{i,t+1} = P_{i,t+1}/((1 - b_i)P_{i,t})$, implying that $\tau_i^s = 1/(1 - b_i)$.[15] Of course, τ_i^ℓ and τ_i^s can be interpreted as any kind of proportional transaction costs associated with long and short positions in the assets. Considering $\widetilde{R}_{t+1}$ as the vector of returns on $2K$ different assets, transaction costs can now be handled by requiring that investors can not go short in the first K assets ($C = \mathbb{R}_+^K$) and can not go long in the last K assets ($C = \mathbb{R}_-^K$). Analogously to the case of short sales constraints, mean-variance efficient portfolios follow from the Kuhn-Tucker conditions of the problem:

$$\begin{aligned} &\max_{\{\widetilde{w}\}} \widetilde{w}' E[\widetilde{R}_{t+1}] - \frac{1}{2}\widetilde{w}' Var[\widetilde{R}_{t+1}]\widetilde{w} \\ \text{s.t. } \widetilde{w}'\iota_{2K} &= 1 \text{ and } \widetilde{w}_i \geq 0,\ \widetilde{w}_{K+i} \leq 0,\ i = 1, 2, .., K, \end{aligned}$$

which are:

$$\begin{aligned} E[\widetilde{R}_{t+1}] - \frac{1}{v}\iota_{2K} + \widetilde{\delta} &= \gamma Var[\widetilde{R}_{t+1}]\widetilde{w}^*, &(4.21)\\ \widetilde{\delta}_i &= 0 \text{ if } \widetilde{w}_i > 0, \\ \widetilde{\delta}_{K+i} &= 0 \text{ if } \widetilde{w}_{K+i} < 0, \\ \widetilde{\delta}_i &\geq 0, \widetilde{\delta}_{K+i} \leq 0,\ i = 1, 2, .., K, \\ \widetilde{\delta}_i \widetilde{w}_i &= 0,\ \forall i. \end{aligned}$$

Let $\widetilde{m}_R(v)_{t+1}$ be the mean-variance stochastic discount factor that prices $\widetilde{R}_{t+1}$ correctly and let $\widetilde{R}_{t+1}^{(v)}$ be the L-dimensional subvector of $\widetilde{R}_{t+1}$ for which the constraints on the short and long positions are not binding. The notation is therefore analogous to the case of short sales constraints

[15] Alternatively, we may also include a bid/ask spread at $t+1$, by letting $\tau_i^\ell = (1-b_i)/(1+a_i)$ and $\tau_i^s = (1+a_i)/(1-b_i)$.

only. The mean-variance stochastic discount factor is now given by:

$$\begin{aligned}\widetilde{m}_R(v)_{t+1} &= v+\widetilde{\alpha}^{(v)\prime}(\widetilde{R}^{(v)}_{t+1}-E[\widetilde{R}^{(v)}_{t+1}]), \\ \widetilde{\alpha}^{(v)} &= Var[\widetilde{R}^{(v)}_{t+1}]^{-1}(\iota_L - vE[\widetilde{R}^{(v)}_{t+1}]),\end{aligned} \tag{4.22}$$

where it is of course again the case that $\widetilde{w}^{(v)} = -\widetilde{\alpha}^{(v)}/\gamma^{(v)}v$, and where $\widetilde{w}^{(v)}$ is the mean-variance efficient portfolio of $\widetilde{R}^{(v)}_{t+1}$ with zero-beta return $1/v$.

In a similar way, we consider long and short positions in the N additional asset as $2N$ different assets. The returns on long position in the additional assets are given by $(\widetilde{r}^{\ell}_{t+1})_k = \tau^{\ell}_k r_{k,t+1}$, $k=1,2,..,N$, while the returns on short positions are given by $(\widetilde{r}^{s}_{t+1})_k = \tau^{s}_k r_{k,t+1}$, $k=1,2,..,N$. The returns on the additional assets are then spanned by the benchmark assets if

$$\begin{aligned}E[\widetilde{m}^{[j]}_R(v)_{t+1}\widetilde{r}^{\ell}_{t+1}] &\leq \iota_N,\ \forall j, \\ E[\widetilde{m}^{[j]}_R(v)_{t+1}\widetilde{r}^{s}_{t+1}] &\geq \iota_N,\ \forall j.\end{aligned} \tag{4.23}$$

As in case of short sales constraints, let $V^{[j]}$ be the set of those values of v for which the subsets of assets for which the constraints on the long and short positions are not binding are the same, with $j=1,2,...,M$. Therefore, we can test for mean-variance spanning of $\widetilde{r}_{t+1}$ by $\widetilde{R}_{t+1}$ by testing whether in the $2M$ regressions

$$\begin{aligned}\widetilde{r}^{\ell}_{t+1} &= a^{[j]}_{\ell} + B^{[j]}_{\ell}\widetilde{R}^{[j]}_{t+1} + \varepsilon^{[j]}_{\ell,t+1}, \\ \widetilde{r}^{s}_{t+1} &= a^{[j]}_{s} + B^{[j]}_{s}\widetilde{R}^{[j]}_{t+1} + \varepsilon^{[j]}_{s,t+1},\end{aligned} \tag{4.24}$$

the following restrictions hold:

$$\begin{aligned}a^{[j]}_{\ell}v^{[j]}_{\min} + B^{[j]}_{\ell}\iota^{[j]} &\leq \iota_N, \\ a^{[j]}_{\ell}v^{[j]}_{\max} + B^{[j]}_{\ell}\iota^{[j]} &\leq \iota_N, \\ a^{[j]}_{s}v^{[j]}_{\min} + B^{[j]}_{s}\iota^{[j]} &\geq \iota_N, \\ a^{[j]}_{s}v^{[j]}_{\max} + B^{[j]}_{s}\iota^{[j]}_{s} &\geq \iota_N,\ \forall j.\end{aligned} \tag{4.25}$$

4.5 Empirical results for emerging markets

In this section we will test whether US-investors that have a well-diversified international stock portfolio can improve upon their efficient set by investing in emerging markets. We use 17 indices from the Emerging Markets Data Base (EMDB) of the International Finance Corporation (IFC). According to the IFC, a country's stock market is an emerging market if that country is classified as either a low- or a middle-income economy by the World Bank, which means that in 1994 the country had to have a per capita GNP of \$8,955 or less. To obtain a sufficiently long data period, monthly observations on the Global Indices are used over the period of January 1985 until June 1996, for six Latin American Countries, seven Asian Countries, one European, one Mideast, and two African countries. Except for Indonesia, Portugal, and Turkey, which are left out of the sample because of many missing observations, these are the same emerging markets as used by Harvey (1995). DeSantis (1994) also uses the emerging markets in our data set, except for Thailand. As noted by Bekaert & Urias (1996), apart from short sales constraints and transaction costs, the returns on the IFC Global Indices may be unattainable to investors because of foreign ownership restrictions, e.g. This problem does not occur with the IFC Investable Indices, which account for these restrictions. The problem with the Investable Indices however, is that there is only a limited sample available. An overview of the available data is given in Appendix 4.C, from which we see that for 9 of the emerging markets in our dataset the Investable Indices are available from January 1989 onwards, while all other indices have a later starting date, which may be as late as November 1993 (Zimbabwe). For Nigeria the Investable Index is not available at all. Notwithstanding this limited availability, some spanning tests will be presented for both the Global and the Investable Indices to show the effect of ownership restrictions. Unless explicitly stated otherwise, the empirical results in this section are based on the IFC Global Indices. The Morgan Stanley Capital International (MSCI) Indices for the USA, Europe and Japan serve as the benchmark assets. Similar indices are also used as reference assets by DeSantis (1994) and Harvey (1995). For all these indices we use (unhedged) monthly holding returns in US dollars. The indices for both the emerging markets and for the developed markets are calculated with dividends reinvested. All data are obtained from Datastream. Some basic summary statistics for net monthly holding returns are given in Table 4.1. Panel A of Table 4.1 provides summary data on the three benchmark indices. Since our test statistics for spanning involve tests for intersection for several values of v, the expectation of the stochastic discount factor, it is useful to restrict the possible range of v beforehand. An upper bound on v is obtained if we do not impose the requirement that investors should invest *all* their wealth in the available assets, but may choose to invest only part of their wealth, i.e., $0 \leq w'\iota \leq 1$ (see also Luttmer (1996)). In effect this allows for the possibility to take long positions in a risk free asset with zero net return (i.e., a return of the form $(P_{t+1} - P_t)/P_t$). This implies that the upper bound for v is 1. If we move upward along the mean-variance frontier, v decreases until $1/v$ equals the intercept of the asymptote of the lines tangent to the mean-variance frontier. This intercept is equal to the expected return on the global minimum variance portfolio, $E[R_{t+1}^{GMV}]$, implying that the lower bound on v is given by $v = 1/E[R_{t+1}^{GMV}]$. Table 4.1 shows that if there are no short sales constraints or transaction costs on the benchmark assets, v is in the range between 0.986 and 1.000. The maximum attainable Sharpe ratios at these boundaries for the benchmark assets are 0.06 and 0.37 respectively. Of course these boundaries have to be adjusted in case there are short sales constraints and/or transaction costs on the benchmark assets R_{t+1}.

Table 4.1: Summary statistics

Panel A provides summary statistics for monthly dollar returns on the MSCI Indices that serve as the benchmark assets. Panel B provides summary statistics for the IFC Emerging Markets Data Base. The sample period is January 1985 until June 1996, giving a total of 138 observations. GMV is the Global Minimum Variance Portfolio. v is the expectation of the stochastic discount factor. "Sh" is the maximum attainable Sharpe ratio.

Panel A: Benchmark indices				
	average	stand.dev.	skewness	kurtosis-3
USAt	1.38%	4.16%	-1.14	6.09
Europe	1.58%	4.91%	-0.56	1.80
Japan	1.43%	7.55%	0.21	0.51
	Correl.			
USA	1.000	0.605	0.211	
Europe		1.000	0.493	
Japan			1.000	
	No frictions			
GMV	1.42%	3.88%		
$v_{\min}$	0.986	Sh($v_{\min}$)	0.055	
$v_{\max}$	1.000	Sh($v_{\max}$)	0.370	
	No short sales allowed			
$v_{\min}$	0.984	Sh($v_{\min}$)	0.086	
$v_{\max}$	1.000	Sh($v_{\max}$)	0.370	
	0.125% transaction costs			
$v_{\min}$	0.990	Sh($v_{\min}$)	0.085	
$v_{\max}$	1.000	Sh($v_{\max}$)	0.338	
	0.50% transaction costs			
$v_{\min}$	0.993	Sh($v_{\min}$)	0.085	
$v_{\max}$	1.000	Sh($v_{\max}$)	0.243	

Table 4.1 also presents summary statistics in case there are short sales constraints on the benchmark indices and in case there exists a transaction cost of 0.125% or 0.5% per month when buying or selling the indices. Although a 0.5% transaction cost is a more realistic estimate of the round trip costs for these benchmark indices than 0.125%, we also consider a 0.125% transaction cost per month to allow for an investment horizon that is longer than one month, thereby decreasing the transaction cost on a monthly basis. It is easy to show that if the proportional transaction cost for a holding period of k months is τ and returns are *i.i.d.* , the mean-variance frontier (portfolios) for this holding period can be obtained from monthly returns with a transaction cost of $\tau^{1/k}$. Therefore,

Table 4.1: Summary statistics (continued)

Panel B: Emerging markets						
	Avg(%)	Std.dev.(%)	v_1	v_2	$m(v_1)(\%)$	$m(v_2)(\%)$
ARG	5.10	27.76	0.999	1.089	1.45	1.42
BRA	3.01	19.23	$--$	$--$	$--$	$--$
CHI	3.38	7.95	0.974	0.880	1.38	1.42
COL	2.83	8.94	0.987	0.946	1.90	1.41
MEX	3.18	13.10	$--$	$--$	$--$	$--$
VEN	2.01	13.81	1.000	0.962	1.45	1.40
IND	1.55	9.80	0.999	0.973	1.45	1.39
KOR	1.64	8.33	1.021	0.925	1.43	1.41
MAL	1.37	7.64	$--$	$--$	$--$	$--$
PAK	1.43	7.09	0.998	0.975	1.46	1.38
PHI	3.41	10.37	0.975	1.059	1.38	1.43
TAI	2.72	14.34	$--$	$--$	$--$	$--$
THA	2.39	8.69	1.005	1.213	1.44	1.42
GRE	2.11	11.81	1.034	0.674	1.43	1.42
JOR	0.64	4.86	1.005	0.986	1.44	2.56
NIG	1.69	15.27	1.061	0.871	1.43	1.42
ZIM	2.51	9.21	0.991	0.957	1.51	1.40

with a four-month holding period, the implied monthly transaction cost is $(1.005)^{\frac{1}{4}} \approx 1.00125$, or 0.125%.

Panel B of Table 4.1 shows some summary statistics for the emerging markets. A quick look at the data reveals that the emerging markets indices are usually much more variable than the benchmark indices, but also have higher average returns. For the monthly holding returns we use, the average standard deviation of the emerging markets indices is 11.66% and the average expected return is 2.41%, compared with 5.54% and 1.46% for the benchmark indices. Table 4.1 also provides some information on the diversification possibilities of each emerging market relative to the three benchmark indices if transaction costs are negligible and short selling is allowed. Since the Wald test-statistic for intersection for a given value of v is a quadratic function of v, we can solve for the range of values of v for which the test statistic is smaller than the α%-critical value. The third and fourth column of Panel B give the (unrestricted) range of v for which the hypothesis that the mean-variance efficient frontier of the three benchmark indices plus the emerging market intersects the mean-variance efficient frontier of the three benchmark indices only, can not be rejected at the 5% statistical significance level. For instance, in case of Argentina, the hypothesis of intersection (neglecting market frictions) can not be rejected at the 5% level if $0.999 \leq v \leq 1.089$. Columns 5 and 6 of Panel B translate these values of v into expected portfolio returns for the three benchmark

Table 4.1: Summary statistics (continued)

Panel C: Summary statistics for subperiods				
	jan-85 - dec-91		jan-92 - jun-96	
	Avg(%)	Std.dev.(%)	Avg(%)	Std.dev.(%)
USA	1.51	5.03	1.17	2.23
EUR	1.96	5.70	1.00	3.28
JAP	1.96	7.96	0.61	6.87
ARG	7.74	34.44	0.98	10.26
BRA	3.01	22.44	3.02	12.96
CHI	4.38	8.08	1.83	7.55
COL	3.65	8.69	1.56	9.24
MEX	4.87	14.14	0.54	10.91
VEN	3.60	13.68	-0.45	13.77
IND	1.83	8.78	1.12	11.27
KOR	2.30	8.71	0.62	7.67
MAL	1.01	7.99	1.93	7.10
PAK	2.23	6.10	0.19	8.32
PHI	4.08	11.43	2.38	8.46
TAI	3.57	15.96	1.40	11.37
THA	2.63	8.68	2.03	8.78
GRE	3.28	14.13	0.30	6.55
JOR	0.59	5.31	0.72	4.09
NIG	0.59	9.99	3.39	21.01
ZIM	3.08	8.83	1.61	9.78

indices. Thus, for investors that initially hold minimum variance portfolios of the three benchmark indices with expected returns that are either below 1.42% per month or above 1.45% per month, the inclusion of Argentina yields a shift in the frontier that is statistically significant at the 5% level. If no bounds are reported in Panel B of Table 4.1, this means that the intersection-test never rejects at the 5% level for that emerging market.

Although these results indicate that most emerging markets can offer significant diversification possibilities, it is not clear whether such diversification benefits are actually attainable. For one thing, it may very well be the case that the diversification benefits offered by the emerging markets can only be realized if short positions are taken in emerging markets, the benchmark assets, or both. Whether or not the shifts in the mean-variance frontiers are statistically significant once short sales constraints are taken into account, will be the subject of the next section.

Finally, Panel C of Table 4.1 shows the average returns and standard deviations for two subperiods. Bekaert et al. (1996) provide ample evidence that especially the behavior of the emerging returns has been changing over time. One important reason for this are the many liberalizations that have taken place in the emerging markets (see, e.g., Bekaert, 1995), causing the emerging markets to become more integrated with the developed markets. As noted by Bekaert et al. (1996), most of the capital market liberalizations in the emerging markets took place before 1992. For this reason we split our sample in a pre-1992 and a post-1992 period. From the average returns and the standard deviations of the returns it is obvious that there are important differences between the pre-1992 and post-1992 period, both for the emerging and for the developed markets. For one thing, the average monthly returns and the standard deviations have decreased in the post-1992 period relative to the pre-1992 period, for both the emerging and the developed markets, although there are also a number of individual emerging markets for which the average returns and/or the standard deviations of the returns have increased in the post-1992 period. The average return for the benchmark indices has decreased from 1.81% per month in the pre-1992 period to 0.92% in the post-1992 period. The relatively high average return in the pre-1992 period is mainly due to the high returns in the first two years of our sample period. For the emerging markets the average returns in the two subperiods are 3.08% and 1.36% per month respectively. The standard deviations likewise decreased over the two subperiods. However, the stylized fact that both the average returns and the volatility in the emerging markets are higher than in the developed markets is present in both the pre-1992 and the post-1992 period. Also, whereas the average correlations between the benchmark returns were 0.46 and 0.34 in the two subperiods, the correlations between the emerging markets and between the emerging markets and the benchmark assets were usually rather low in both subperiods, despite the liberalizations of the emerging markets. Therefore, although the return characteristics for the emerging markets may have changed after the liberalizations, these stylized facts suggest that in the post-1992 period there may still be diversification benefits from including emerging markets in a portfolio of the benchmark assets considered here.

4.5.1 Results for spanning tests with short sales constraints

The analysis in the previous section already suggested that, in the absence of market frictions, many emerging markets yield significant diversification benefits relative to the benchmark indices for the US, Europe, and the world. Table 4.2 shows Wald test-statistics for the hypothesis that the returns on these three indices span the returns for each emerging market. In this table and the following, the emerging markets are organized according to their geographical region: Latin America, Asia, and "Other". For each group, the first line shows the spanning test-statistic and the associated p-value in case there are no short sales restrictions on either the benchmark assets or the emerging markets. In this case, the hypothesis of mean-variance spanning is readily rejected at the 5% significance

Table 4.2: Spanning tests with short sales constraints

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI Indices for the USA, Europe and Japan. The numbers in the table are Wald test statistics. The numbers in parentheses are p-values associated with the Wald test statistics. The tests are based on monthly returns for the period January 1985 until June 1996. The results for the IFC Investable Indices are for January 1989 until June 1996, or for a shorter period if no data for the IFC Investable Index was available. NA = not available.

Latin America								
	Arg	Bra	Chi	Col	Mex	Ven		All
	No restrictions							
Wald	4.43	1.30	17.54	21.11	3.31	18.40		61.07
(p)	(0.109)	(0.522)	(0.000)	(0.000)	(0.192)	(0.000)		(0.000)
	No short sales of emerging markets							
Wald	3.72	1.22	14.46	9.95	3.08	3.72		31.44
(p)	(0.032)	(0.157)	(0.001)	(0.003)	(0.053)	(0.035)		(0.000)
	No short sales							
Wald	3.70	1.21	14.36	9.88	3.23	3.69		31.74
(p)	(0.075)	(0.313)	(0.000)	(0.003)	(0.109)	(0.070)		(0.000)
	Investable indices, no short sales							
Wald	2.01	2.11	6.74	4.42	1.65	5.46		14.81
(p)	(0.181)	(0.175)	(0.012)	(0.057)	(0.219)	(0.024)		(0.027)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	
	No restrictions							
Wald	32.50	6.11	0.74	44.07	6.05	2.36	4.60	86.16
(p)	(0.000)	(0.047)	(0.689)	(0.000)	(0.048)	(0.307)	(0.100)	(0.000)
	No short sales of emerging markets							
Wald	3.92	1.47	0.05	4.71	6.05	1.87	3.28	14.82
(p)	(0.029)	(0.148)	(0.481)	(0.017)	(0.011)	(0.098)	(0.044)	(0.004)
	No short sales							
Wald	3.89	1.46	0.05	4.68	6.00	1.86	3.25	14.71
(p)	(0.069)	(0.256)	(0.680)	(0.042)	(0.017)	(0.194)	(0.082)	(0.029)
	Investable indices, no short sales							
Wald	0.21	0.17	0.97	2.04	0.88	0.67	0.61	3.93
(p)	(0.614)	(0.627)	(0.329)	(0.196)	(0.366)	(0.398)	(0.429)	(0.578)

level for 9 out of the 17 emerging markets. A joint test for spanning of all the emerging markets in a geographical group always rejects the null hypothesis of spanning. These results confirm the findings of, e.g., DeSantis (1994) and Harvey (1995), which were discussed in Section 4.7 of Chapter 2. As noted before however, these diversification benefits may not be attainable to investors, since they may require short selling of the emerging markets indices, the benchmark indices, or both.

Table 4.2: Spanning tests with short sales constraints (continued)

Other					
	Gre	Jor	Nig	Zim	All
	No restrictions				
Wald	4.28	76.37	4.29	24.29	105.07
(p)	(0.118)	(0.000)	(0.117)	(0.000)	(0.000)
	No short sales of emerging markets				
Wald	1.59	1.09	0.83	8.38	11.16
(p)	(0.125)	(0.186)	(0.227)	(0.001)	(0.013)
	No short sales				
Wald	1.58	1.08	0.83	8.31	11.08
(p)	(0.227)	(0.312)	(0.387)	(0.006)	(0.036)
	Investable indices, no short sales				
Wald	1.59	1.02	NA	3.21	3.25
(p)	(0.240)	(0.334)	NA	(0.105)	(0.391)

If investors are not allowed to go short in the emerging markets, while still retaining the possibility to sell the benchmark indices short, the main conclusion does not change. The second line for each geographical group in Table 4.2 shows that there are now 10 out of 17 rejections at the 5% significance level. Notice that the rejections that are found do not always coincide with a rejection in the no-friction case. Taking into account short sales constraints on the emerging markets causes decreases in the Wald test-statistic that are often nontrivial. When performing a joint spanning test for all emerging markets within a geographical group, the effect of short sales constraints is strongest for the group "Other". However, the hypothesis of spanning can always be rejected at the 5% significance level, reflecting the fact that the short sales constraints on the emerging markets are usually not binding. Because of the high average returns in the emerging markets, investors with low risk aversions can benefit from buying the emerging markets asset and selling (part of) their benchmark assets.

It may be the case though, that investing in the emerging markets only extends the efficient set when the portfolio of the benchmark assets already contains short positions. In order to account for short sales restrictions on the benchmark assets as well, Table 4.2 also presents spanning tests in case there are short sales restrictions on both the emerging markets and the benchmark assets. These results are presented in the third line for each geographical group in Table 4.2. The effect of short sales restrictions is more pronounced in this case. If investors are not allowed to short sell any of the assets, the hypothesis of spanning can be rejected at the 5% significance level for only 5 markets: Chile, Colombia, Pakistan, the Philippines, and Zimbabwe. Joint tests for each geographical group always reject the hypothesis of spanning however. The results in Table 4.2

therefore show that our benchmark investors can still benefit from investing in those markets, even though there may be short sales constraints.

Even for the five emerging markets for which the null hypothesis is rejected, the diversification benefits may not be attainable however, because of foreign ownership restrictions. Bekaert (1995) discusses several measures of the extent of foreign ownership restrictions in emerging markets. One such measure, for instance, is the ratio of the IFC Investable Index over the IFC Global Index, since the Investable Index takes into account foreign ownership restrictions on each stock traded in an emerging markets. Except for Colombia, Bekaert (1995) reports ratios that are rather low for these five countries (in particular for Zimbabwe). Thus, except possibly for Colombia, the diversification benefits suggested by Table 4.2 may be difficult or impossible to obtain.

To shed some further light on this issue, the last line for each geographical group in Table 4.2 gives the results for the spanning tests in case the IFC Investable Indices are used instead of the Global Indices. The null hypothesis is again whether the emerging market indices are spanned by the benchmark assets in case there are short sales constraints on both the emerging markets and the benchmark assets. For three of the five markets just mentioned, Pakistan, the Philippines and Zimbabwe, the hypothesis of spanning can not be rejected for the Investable Indices, suggesting that the ownership restrictions are indeed binding for these countries. For Colombia the hypothesis of spanning can be rejected at the 10% level, but it is only in case of Chile that we can still reject spanning at the 5% level. Joint tests for all emerging markets within a geographical group reject the hypothesis of spanning only for Latin America.

Summarizing, it is clear that the hypothesis of mean-variance spanning is easily rejected in case there are no market frictions. In case there are short sales constraints there is still a lot of evidence against this hypothesis. Especially when there are only short sales constraints on the emerging markets but not on the benchmark indices, the shifts from the mean-variance frontiers of the benchmark indices to the frontiers of the benchmark indices plus the emerging markets are often statistically significant. The number of countries for which the hypothesis of spanning can be rejected is much smaller once there are also short sales constraints on the benchmark indices, although the joint tests for each geographical still reject the hypothesis of spanning in all cases. However, the countries for which the rejections remain significant even after allowing for short sales constraints, seem to be countries for which ownership restrictions are particularly severe. Taking into account ownership restrictions as well as short sales constraints, the hypothesis of spanning can only be rejected for Latin America.

4.5.2 Results for spanning tests with transaction costs

In this section we consider the effects of transaction costs on the hypothesis that the mean-variance frontier of the benchmark indices spans the frontiers of the benchmark indices plus the individual emerging markets. We assume that investors have to pay a transaction cost of either 0.125% or 0.5% per transaction when buying or (short) selling the benchmark assets. Notice that the proportional transaction costs considered here can be interpreted as a round trip cost. Although 0.5% may be a more realistic estimate of the round trip transaction costs for the benchmark indices, observe that since we use monthly returns this implicitly assumes that trading takes place once a month. The effect of transaction costs as high as 0.5% may be particularly severe with this rather high trading frequency. Therefore, to mitigate this effect, we also allow for a 0.125% transaction cost, which may be a more realistic estimate for investors who have an investment horizon of, say, four months. As already noted in the discussion of Table 4.1, with a 0.5% transaction cost per month on the benchmark assets, v is in the range between 1.000 and 0.993 as shown in Table 4.1, where investors want to take long positions in the MSCI Indices for the USA, Europe and Japan, or in the indices for the USA and Europe only. A transaction cost of either 0.125% or 0.5% per month precludes investors from taking any short position in the benchmark indices.

With a 0.125% transaction cost per month, v is in the range between 1.000 and 0.990, and investors also want to take long positions in the USA, Europe and Japan index or in the USA and Europe index only. As noted by Bekaert & Urias (1996), the IFC Indices for the emerging markets are characterized by high transaction costs and other market frictions. Therefore, we consider the effect of increasing the transaction costs on those indices to a level as high as eight times the level for the benchmark indices.

The results for the spanning tests with a 0.125% transaction cost on the benchmark assets and an increasing transaction cost on the emerging markets are presented in Table 4.3. Here the null hypothesis is that long positions in each emerging market are spanned by long positions in the three mature market indices. To put things into perspective, the results in Table 4.3 should be compared with the results in the third line of each geographic group in Table 4.2, where there are short sales restrictions on the emerging markets as well as on the benchmark assets. The first line of each geographic group in Table 4.3 shows the results if there is a 0.125% transaction cost on both the benchmark assets and the emerging markets. A quick look at Table 4.2 and 4.3 shows that the effect of a 0.125% transaction in itself is not very dramatic, since the values of the test statistics and the associated p-values are roughly of the same order of magnitude in the two tables. In case of a 0.125% transaction cost, the hypothesis of spanning can be rejected at the 5% level for 9 emerging markets. The joint tests also reject the hypothesis of spanning for each geographical region at the 5% level.

Table 4.3: Spanning tests with transaction costs

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI Indices for the USA, Europe and Japan when there are transaction costs. The table assumes that there is a 0.125 percent transaction cost on the benchmark assets. The numbers in the table are Wald test statistics. The numbers in parentheses are the p-values associated with the Wald test statistics. The tests are based on monthly returns for January 1985 until June 1996.

Latin America								
	Arg	Bra	Chi	Col	Mex	Ven		All
	0.125 % tr. cst. on emerging markets							
Wald	3.60	1.19	13.97	9.28	2.89	3.28		29.74
(p)	(0.032)	(0.169)	(0.000)	(0.002)	(0.063)	(0.042)		(0.000)
	0.250 % tr. cst. on emerging markets							
Wald	3.42	1.04	12.67	8.37	2.50	2.93		26.99
(p)	(0.040)	(0.176)	(0.001)	(0.002)	(0.069)	(0.051)		(0.000)
	0.500% tr. cst. on emerging markets							
Wald	3.06	0.77	10.26	6.70	1.81	2.29		21.93
(p)	(0.043)	(0.226)	(0.001)	(0.009)	(0.112)	(0.080)		(0.000)
	1.000 % tr. cst. on emerging markets							
Wald	2.41	0.35	6.20	3.91	0.76	1.25		13.51
(p)	(0.068)	(0.323)	(0.011)	(0.036)	(0.226)	(0.142)		(0.012)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
	0.125 % tr. cst. on emerging markets							
Wald	3.31	1.31	0.04	3.93	5.98	1.80	3.13	13.43
(p)	(0.044)	(0.154)	(0.456)	(0.029)	(0.012)	(0.105)	(0.049)	(0.011)
	0.250 % tr. cst. on emerging markets							
Wald	2.80	0.94	0.00	3.19	5.33	1.55	2.57	11.38
(p)	(0.065)	(0.194)	(0.944)	(0.052)	(0.015)	(0.128)	(0.065)	(0.020)
	0.500 % tr. cst. on emerging markets							
Wald	1.92	0.39	0.00	1.94	4.13	1.11	1.62	7.90
(p)	(0.103)	(0.325)	(0.789)	(0.105)	(0.029)	(0.162)	(0.127)	(0.086)
	1.000 % tr. cst. on emerging markets							
Wald	0.65	0.00	0.00	0.36	2.19	0.44	0.37	3.23
(p)	(0.250)	(0.803)	(0.637)	(0.314)	(0.075)	(0.291)	(0.320)	(0.408)

The most interesting result of Table 4.3 is perhaps the effect of an increase in the transaction cost on the emerging markets, while keeping the transaction cost for the benchmark assets at 0.125%. Doubling the transaction cost on the emerging markets to 0.250% has only a minor effect on the individual markets. However, an increase to 0.5% per month leaves us with only 5 rejections at the

Table 4.3: Spanning tests with transaction costs (continued)

Other					
	Gre	Jor	Nig	Zim	All
	0.125 % tr. cst. on emerging markets				
Wald	1.46	0.62	0.73	7.66	9.85
(p)	(0.119)	(0.269)	(0.242)	(0.006)	(0.022)
	0.250 % tr. cst. on emerging markets				
Wald	1.19	0.25	0.59	6.86	8.40
(p)	(0.158)	(0.344)	(0.263)	(0.005)	(0.027)
	0.500 % tr. cst. on emerging markets				
Wald	0.73	0.00	0.34	5.39	6.18
(p)	(0.215)	(0.547)	(0.314)	(0.013)	(0.080)
	1.000 % tr. cst. on emerging markets				
Wald	0.15	0.00	0.05	2.98	3.10
(p)	(0.386)	(0.539)	(0.478)	(0.057)	(0.280)

5% level, while the joint tests only reject the null hypothesis for Latin America. The rejections at the 5% level are for Argentina, Chile, Colombia, the Philippines and Zimbabwe. As noted in the previous section, except for Colombia, these are markets in which foreign ownership restrictions probably prevent investors to realize the diversification benefits that are potentially offered by these emerging markets. However, even with a transaction cost as high as 1.0% on the emerging markets, i.e., 8 times as high as for the benchmark assets, there is still some evidence against the hypothesis of spanning for both Chile and Colombia, as well as for all of the Latin American countries together. Nonetheless, for the bulk of the emerging markets, increasing the transaction costs leaves us with little evidence in favor of diversification benefits. Notice though, that these are the transaction costs that investors have to pay when they trade their portfolio every month. If the round trip cost for emerging markets is in the order of magnitude of, say, 0.5%, then the results in Table 4.3 suggest that investing in emerging markets is worthwhile if investors trade once every two months or less.

These results are confirmed by the results in Table 4.4, where the effect of transaction costs is shown for the Investable Indices. As in Table 4.3, in Table 4.4 it is assumed that there is a 0.125% transaction cost on the benchmark assets, and there are two levels of transaction costs for the emerging markets: 0.125% and 0.5% respectively. With a 0.125% transaction costs it is only for some Latin American countries that the hypothesis of spanning can be rejected. Joint tests for the three geographical groups also only reject for Latin America. In case of a 0.5% transaction costs there are still rejections at the 5% level for the Latin American countries. Therefore, as with

Table 4.4: Testing for spanning with transaction costs: Investable indices

The table presents test results for the hypothesis that there is mean-variance spanning of the IFC Investable Indices for the emerging markets by three benchmark assets, which are the MSCI Indices for the USA, Europe and Japan when there are transaction costs. The table assumes that there is a 0.125 percent transaction cost on the benchmark assets. The numbers in the table are Wald test statistics. The numbers in parentheses are the p-values associated with the Wald test statistics. The tests are based on monthly returns for January 1989 until June 1996, or on a shorter period if no data for the IFC Investable Index was available. NA = not available.

Latin America								
	Arg	Bra	Chi	Col	Mex	Ven		All
	0.125 % tr. cst. on emerging markets							
Wald	2.17	1.80	6.47	4.31	1.65	5.10		9.72
(p)	(0.099)	(0.129)	(0.012)	(0.023)	(0.137)	(0.022)		(0.040)
	0.500 % tr. cst. on emerging markets							
Wald	1.82	1.40	4.46	3.38	0.88	4.42		7.12
(p)	(0.114)	(0.146)	(0.023)	(0.040)	(0.235)	(0.030)		(0.102)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
	0.125 % tr. cst. on emerging markets							
Wald	0.15	0.10	0.96	1.88	0.43	0.62	0.74	2.73
(p)	(0.453)	(0.487)	(0.215)	(0.115)	(0.321)	(0.279)	(0.248)	(0.438)
	0.500 % tr. cst. on emerging markets							
Wald	0.02	0.00	0.24	1.29	0.09	0.29	0.20	1.92
(p)	(0.566)	(0.652)	(0.369)	(0.178)	(0.429)	(0.388)	(0.371)	(0.584)
Other								
	Gre	Jor	Nig	Zim				All
	0.125 % tr. cst. on emerging markets							
Wald	1.49	0.79	NA	3.03				3.06
(p)	(0.141)	(0.233)	NA	(0.057)				(0.216)
	0.500 % tr. cst. on emerging markets							
Wald	0.93	0.08	NA	2.42				2.42
(p)	(0.221)	(0.447)	NA	(0.092)				(0.274)

the short sales constraints only, in case of transaction costs the results in Table 4.3 and 4.4 show that spanning can only be rejected for Latin America.

Finally, Table 4.5 gives some idea of the transaction costs that are needed to keep investors out of the emerging markets. Starting with a round trip cost of 0.5% for the benchmark assets and assuming monthly trading, Table 4.5 presents levels of transaction costs in the emerging markets above which the hypothesis of spanning can not be rejected at the 5% and 10% level respectively. For instance, in case of Argentina a round trip cost below 1.50% is needed to reject spanning by the benchmark assets at the 10% level and a round trip cost below 0.60% is needed to reject spanning at the 5% level. The estimates of 0.00% in case of Brazil for instance, imply that spanning can never be rejected at the 10% level, no matter how low the transaction costs are. The estimates in

Table 4.5: Transaction cost bounds

The table presents estimated transaction cost bounds for the emerging markets in order to reject spanning of each emerging market by three benchmark assets at the 5% and 10% significance level. The three benchmark assets are the MSCI Indices for the USA, Europe and Japan. The table assumes that there is a 0.5 percent transaction cost on the benchmark assets. The estimated transaction costs are in percentages per month. All results are based on monthly returns for January 1985 until June 1996. The actual transaction costs are from Baring Securities as reported by Bekaert et al. (1996). NA = not available.

Latin America							
	ARG	BRA	CHI	COL	MEX	VEN	
10%-bound	1.50	0.00	1.70	1.50	0.30	0.70	
5%-bound	0.60	0.00	1.50	1.20	0.00	0.30	
actual tr.cst.	1.55	0.85	3.93	1.00	0.93	NA	
Asia							
	IND	KOR	MAL	PAK	PHI	TAI	THA
10%-bound	0.40	0.20	0.00	0.50	1.40	0.30	0.75
5%-bound	0.10	0.00	0.00	0.30	1.10	0.00	0.40
actual tr.cst.	1.50	NA	0.69	0.38	0.94	0.47	0.70
Other							
	GRE	JOR	NIG	ZIM			
10%-bound	0.20	0.00	0.00	1.40			
5%-bound	0.00	0.00	0.00	0.10			
actual tr.cst.	0.48	0.58	NA	NA			

Table 4.5 suggest that with a 0.5% round trip costs on the benchmark assets, transaction costs for the emerging markets need not be particularly high to keep investors out of these markets. It is only in a few cases that a transaction cost of at least two times the level in the benchmark assets is needed to keep investors out of the market. (Admittedly this is a rather aggressive interpretation of the results in Table 4.5, since the fact that we can not reject the hypothesis of spanning by no means implies that there is spanning.) Once more, if the hypothesis of spanning is not to be rejected, a transaction cost more than two times the one for the benchmark assets is needed for only three markets: Chile, Colombia, and the Philippines.

To get some further intuition about the importance of these transaction costs, the third line for each geographic group in Table 4.5 gives an estimate of the actual round trip costs in the emerging markets. These estimates are from Barings Securities and reported by Bekaert et al. (1996). The reported transaction costs are calculated from the percentage spread, which is the difference between the offer and bid price divided by the average of the offer and bid price for a security. To obtain a spread for each country, the percentage spreads of individual stocks are weighted by the capitalization of each stock within each country (see Bekaert et al. (1996)).

Interestingly, except for Colombia and the Philippines, the actual transaction costs are always higher than the calculated 5%-bounds in Table 4.5. Even for Colombia and the Philippines the actual transaction is rather high compared with the other markets, and is close to the estimated 5%-bound. Also, both the 5%-bound and the actual transaction cost are highest for the same market: Chile. Taken together, the evidence presented in this section suggests that the individual emerging markets are spanned by the three benchmark indices when allowing for transaction costs. This conclusion is based on investors that trade their portfolio on a monthly basis however. For investors that trade their portfolio less frequently there is still evidence that there are diversification benefits from investing in emerging markets, even after transaction costs.

4.5.3 Spanning tests for the post-liberalization periods

As already suggested by the summary statistics in Panel C of Table 4.1 and by previous studies (e.g., Bekaert (1995)), the liberalizations that have taken place in many emerging markets may have altered the return distributions in those markets in a nontrivial way. To see the effects of these liberalizations on some of our results we repeat the spanning tests for the no-frictions case and for the case where there are short sales constraints, for the periods after major liberalizations of the emerging markets. Appendix 4.C provides the last major liberalization date for each emerging stock market as reported by Bekaert (1995). Starting from the month after this liberalization until the end of our sample period, we repeat the analysis in Table 4.2 for each emerging market. The results for these subperiods are presented in Table 4.6. For each geographical group of emerging markets, the last column of Table 4.6 similarly presents joint spanning tests for the period from the last liberalization in the geographical group until the end of the sample period. Since there are no major liberalizations for the group "Other", we do no report results for this group in Table 4.6.

Spanning tests in case there are no market frictions are presented in the first row of each geographical group in Table 4.6. Except for Colombia and Thailand, the hypothesis of spanning can not be rejected for any of the emerging markets in Latin America and Asia at the 10% level. Joint tests for all emerging markets within each geographical group still reject the null hypothesis at for all geographical groups. Thus, for the post-liberalization period, there is much less evidence against the hypothesis that there is mean-variance spanning, even in case there are no frictions.

This is also the case for the remainder of Table 4.6, which shows the test statistics in case there are short sales constraints. In case there are short sales constraints on all assets the hypothesis of spanning can only be rejected at the 5% level for Colombia. The joint tests for the emerging markets in Latin America and in Asia never reject the null hypothesis. These results also hold true when the Investable Indices are used instead of the Global Indices. Therefore, whereas the hypothesis of spanning is strongly rejected when data for the whole sample period are used and

Table 4.6: Spanning test for the post-liberalization periods

The table presents test results for the hypothesis that there is mean-variance spanning of emerging markets by three benchmark assets, which are the MSCI Indices for the USA, Europe and Japan, after liberalizations in the emerging markets have taken place. For each emerging market results are shown for the period after liberalization of the stock market has taken place, as reported in Appendix 4.C.. If there is no liberalization during the sample period, the whole sample period is used. The numbers in the table are Wald test statistics. The numbers in parentheses are p-values associated with the Wald test statistics. The tests are based on monthly returns for the month after liberalization (or from January 1985) until June 1996.

Latin America								
	Arg	Bra	Chi	Col	Mex	Ven		All
	No restrictions							
Wald	2.12	2.28	4.38	5.14	1.02	1.85		18.12
(p)	(0.347)	(0.320)	(0.112)	(0.077)	(0.599)	(0.397)		(0.000)
	No short sales of emerging markets							
Wald	0.76	0.85	3.52	4.01	0.93	0.01		5.60
(p)	(0.221)	(0.242)	(0.036)	(0.031)	(0.201)	(0.524)		(0.163)
	No short sales							
Wald	0.27	0.40	3.51	4.01	0.85	0.01		5.03
(p)	(0.606)	(0.298)	(0.061)	(0.026)	(0.194)	(0.480)		(0.148)
	Investable indices, no short sales							
Wald	0.27	0.38	3.94	4.43	0.95	3.78		5.57
(p)	(0.603)	(0.301)	(0.047)	(0.023)	(0.170)	(0.026)		(0.127)
Asia								
	Ind	Kor	Mal	Pak	Phi	Tai	Tha	All
	No restrictions							
Wald	3.16	3.33	0.74	4.42	3.19	1.31	4.60	16.50
(p)	(0.206)	(0.189)	(0.689)	(0.110)	(0.203)	(0.520)	(0.100)	(0.000)
	No short sales of emerging markets							
Wald	0.12	0.12	0.05	0.195	2.72	0.70	3.28	1.33
(p)	(0.433)	(0.436)	(0.481)	(0.112)	(0.065)	(0.249)	(0.044)	(0.633)
	No short sales							
Wald	0.12	0.13	0.05	1.67	2.67	0.55	3.25	1.30
(p)	(0.451)	(0.404)	(0.680)	(0.094)	(0.068)	(0.226)	(0.082)	(0.655)
	Investable indices, no short sales							
Wald	0.21	0.18	0.97	2.03	1.63	0.53	0.61	3.11
(p)	(0.403)	(0.369)	(0.329)	(0.088)	(0.114)	(0.236)	(0.429)	(0.362)

there are no market frictions, there is hardly any evidence against spanning, with or without market frictions, for the subperiods after liberalization of the emerging markets.

4.6 Concluding remarks

There is substantial evidence available in the literature that suggests that, in the absence of market frictions, US-investors can benefit from including emerging markets assets in their well-diversified international portfolio of developed market assets. We try to shed some further light on this issue by testing whether emerging market indices are spanned by developed market indices when investors face short sales constraints and/or transaction costs. There are still some open issues that have not been considered in this chapter and that are left for future research. For instance, except for splitting the sample according to market liberalizations, our results do not account for time variation in expected returns and volatilities. Previous studies suggest that there is such time variation, and that it is important to consider dynamic trading strategies. Future research plans to take these issues into account as well.

Appendix 4.A Duality between mean-variance frontiers and volatility bounds with short sales constraints

In this appendix we show that the duality between mean-variance frontiers and volatility bounds still holds when there are short sales constraints on the assets. In particular, we show that the stochastic discount factor with expectation $v > 0$, that has the lowest variance among all stochastic discount factors that have expectation v and that price the returns R_{t+1} correctly subject to short sales constraints, is a linear function of the return on a mean-variance optimal portfolio with zero-beta rate $1/v$, subject to short sales constraints. With short sales constraints on the K assets with return vector R_{t+1} the set of returns available to investors is given by:

$$X^p = \{R^p_{t+1} : R^p_{t+1} = w' R_{t+1}, w \geq 0 \text{ and } w'\iota_K = 1\}.$$

Valid stochastic discount factors M_{t+1} satisfy:

$$E[M_{t+1} R_{t+1}] \leq \iota_K, \tag{A.1}$$

where there are strict equalities for the assets for which the short sales constraints are not binding (otherwise the agent with a utility function corresponding to M_{t+1} would sell part of his holding of $R_{i,t+1}$ until an equality is obtained). Recall that M_{t+1} is proportional to the derivative of an agents derived utility of wealth function, given his optimal portfolio choice, w^*. Let $u(w' R_{t+1})$ be a derived utility of wealth function (strictly increasing and concave). The problem that the agent has to solve is

$$\max_{\{w\}} E[u(w' R_{t+1})] - \eta(w'\iota_K - 1) + w'\delta,$$

where δ is the K-dimensional vector of Kuhn-Tucker multipliers for the condition that $w \geq 0$. The first order conditions of the optimization problem imply

$$\begin{aligned} E[u'(w^{*\prime} R_{t+1}) R_{t+1}] - \eta\iota_K + \delta &= 0, \\ w^{*\prime}\delta &= 0, \\ \delta_i &= 0 \text{ if } w_i > 0 \\ \delta_i &\geq 0, \forall i, \end{aligned} \tag{A.2}$$

implying that $M_{t+1} = u'(w^{*\prime} R_{t+1})/\eta$ is a valid stochastic discount factor.

Notice that the first order conditions imply that

$$E[M_{t+1} w^{*\prime} R_{t+1}] = 1. \tag{A.3}$$

Let X^* be the set of returns on optimal portfolios subject to short sales constraints:

$$\begin{aligned} X^* &= \{R^p_{t+1} : R^p_{t+1} = w^{*\prime} R_{t+1}, w^* \geq 0, w^{*\prime}\iota_K = 1, \text{ and} \\ \exists M_{t+1} \text{ s.t. } E[M_{t+1}R_{t+1}] &\leq \iota_K \text{ and } E[M_{t+1}w^{*\prime}R_{t+1}] = 1\}, \end{aligned}$$

and observe that $X^* \subset X$.

For a stochastic discount factor with expectation v, define excess returns $R^e_{t+1} \equiv R^p_{t+1} - 1/v$. Using obvious notation it follows that for $R^p_{t+1} \in X^*$,

$$\begin{aligned} 0 &= E[M(v)_{t+1}R^e_{t+1}] = vE[R^e_{t+1}] + \rho_{RM}\sigma_R\sigma_M \\ \Rightarrow & \left|\frac{E[R^e_{t+1}]}{\sigma_R}\right| \leq \frac{\sigma_M}{v}. \end{aligned} \tag{A.4}$$

Thus, the maximum (absolute value of the) Sharpe ratio that can be obtained from the set of optimal portfolio returns, X^*, gives a lower bound on the volatility of admissable stochastic discount factors with expectation v (see, e.g., Hansen & Jagannathan (1991)).

First consider the returns that are in X, i.e., the set of all possible portfolio returns subject to short sales constraints. The set of mean-variance efficient portfolios is characterized by (5):

$$\begin{aligned} E[R_{t+1}] - \eta\iota_K + \delta &= \gamma Var[R_{t+1}]w^*, \\ \delta_i &= 0 \text{ if } w_i > 0, \\ \delta_i &\geq 0\ \forall i. \end{aligned} \tag{A.5}$$

Now take the mean-variance efficient portfolio for which $\eta = 1/v$. Denoting by $R^{(v)}_{t+1}$ the L-dimensional subvector of R_{t+1} that only contains the returns of the assets for which the short sales constraints in (A.5) are not binding, it is straightforward to show that the mean-variance efficient portfolio in (A.5) is equal to the mean-variance efficient portfolio without short sales constraints of the assets in $R^{(v)}_{t+1}$ only:

$$\begin{aligned} E[R^{(v)}_{t+1}] - \frac{1}{v}\iota_L &= \gamma^{(v)} Var[R^{(v)}_{t+1}]w^{(v)} \text{ and} \\ E[R_{t+1}] - \frac{1}{v}\iota_K + \delta &= \gamma^{(v)} Cov[R_{t+1}, R^{(v)}_{t+1}]w^{(v)}, \end{aligned} \tag{A.6}$$

where $Cov[R_{t+1}, R^{(v)}_{t+1}]$ is de KxL-dimensional covariance matrix of R_{t+1} and its subvector $R^{(v)}_{t+1}$, and $\gamma^{(v)} = (w^{(v)\prime}E[R^{(v)}_{t+1}] - 1/v)/(w^{(v)\prime}Var[R^{(v)}_{t+1}]w^{(v)})$. The maximum Sharpe ratio is therefore equal to

$$\{(E[R^{(v)}_{t+1}] - \frac{1}{v}\iota_L)' Var[R^{(v)}_{t+1}]^{-1}(E[R^{(v)}_{t+1}] - \frac{1}{v}\iota_L)\}^{\frac{1}{2}}. \tag{A.7}$$

Since this is the maximum Sharpe ratio that is attainable over all feasible portfolio returns, equation (A.7) gives a lower bound on the volatility of all admissible stochastic discount factors with expectation v. We can go one step further however, since this lower bound is actually attained by the stochastic discount factor that is linear in the asset returns $R_{t+1}^{(v)}$:

$$\begin{aligned} m_R(v)_{t+1} &= v + \alpha^{(v)\prime}(R_{t+1}^{(v)} - E[R_{t+1}^{(v)}]), \\ \alpha^{(v)} &= Var[R_{t+1}^{(v)}]^{-1}(\iota_L - vE[R_{t+1}^{(v)}]). \end{aligned} \tag{A.8}$$

Since the portfolio $w^{(v)}$ in (A.6) with $\eta = 1/v$ is given by

$$w^{(v)} = \frac{1}{\gamma} Var[R_{t+1}^{(v)}]^{-1}(E[R_{t+1}] - \frac{1}{v}\iota_L) = -\frac{1}{v\gamma}\alpha^{(v)},$$

we have, by using (4.4) that

$$E[m_R(v)_{t+1}R_{t+1}] = vE[R_{t+1}] - v\gamma Cov[R_{t+1}, R_{t+1}^{(v)}]w^{(v)} = \iota_K - v\delta \leq \iota_K, \tag{A.9}$$

if $v > 0$.

Thus, the stochastic discount factor in (A.8) satisfies (2.1), implying that the portfolio return $w^{(v)\prime}R_{t+1}^{(v)}$ that maximizes the Sharpe ratio over all returns in X, is also in X^*. Therefore, $m_R(v)_{t+1}$ is a stochastic discount factor that attains the volatility bound. This is straightforward, since, using (A.8),

$$\frac{\sigma[m_R(v)_{t+1}]}{v} = \{(\frac{1}{v}\iota_L - E[R_{t+1}^{(v)}])' Var[R_{t+1}^{(v)}]^{-1}(\frac{1}{v}\iota_L - E[R_{t+1}^{(v)}])\}^{\frac{1}{2}},$$

which is equal to the maximum Sharpe ratio in (A.7).

Appendix 4.B Proof of the validity of the test

In this appendix we prove a simple but useful lemma. This lemma shows that the fact that we possibly use the incorrect regressions in our spanning and intersection tests (due to sample variation) is asymptotically negligible. Short sales restrictions on the benchmark assets are handled by testing for spanning and intersection on subsets of the available assets, where there is only a finite number of such subsets. The probability of choosing the right subsets tends to one, and this turns out to be a sufficient condition for the validity of the tests. Suppose that we are given a finite number of Wald test-statistics, $\xi(v^{[1]})_T$, ..., $\xi(v^{[M]})_T$, as defined in (4.12), where T is the sample size. Let the space of all possible values of v be partitioned in $V^{[1]}$, ..., $V^{[M]}$, with the interpretation that, depending on the value of the parameter v, one of the test statistics $\xi(v^{[1]})_T$ has desirable properties. Let j indicate the set $V^{[j]}$ to which $v^{[j]}$ belongs. If v_0 denotes the true value of v, one would like to use the test $\xi(v_0)_T$ of course, but this is not possible, since v_0 is unknown. Assume however, that we are given a parameter estimate $\widehat{v}_T$, such that, under v_0

$$\Pr\{\widehat{v}_T \in V^{[j]}\} \to 1, \; T \to \infty.$$

Now we have the following result:

Lemma 1 *For each $c \in I\!R$, we have*

$$\lim_{T\to\infty} \Pr\{\xi(\widehat{v}_T)_T \leq c\} - \Pr\{\xi(v_0)_T \leq c\} = 0.$$

Proof. The proof is very straightforward, using:

$$\begin{aligned}
\Pr\{\xi(\widehat{v}_T)_T &\leq c\} = \sum_{j=1}^{M} \Pr\{\xi(v^{[j]})_T \leq c \text{ and } v^{[j]} = \widehat{v}_T\} = \\
\Pr\{\xi(v_0)_T &\leq c\} - \Pr\{\xi(v_0)_T \leq c \text{ and } v \neq v_0\} + \\
\sum_{v \neq v_0} \Pr\{\xi(v)_T &\leq c \text{ and } v = \widehat{v}_T\},
\end{aligned}$$

and that the latter two terms converge to zero. ■

Appendix 4.C Available data for the emerging markets

This appendix describes some characteristics of the data that are used in this chapter. The table gives the first month that the IFC Investable Indices for the emerging markets used in this paper appear in the sample. The Global Indices are always available from January 1985 onwards. The sample period ends in June 1996. The last column of the table contains the last major liberalization date of the emerging stock market, based on Bekaert (1995).

Country	*Code*	*Starting Date IFC Investable*	*Last major liberalization*
Argentina	ARG	Jan 1989	Dec 1989
Brazil	BRA	Jan 1989	Jul 1991
Chile	CHI	Jan 1989	Apr 1990
Colombia	COL	Mar 1991	Feb 1991
Mexico	MEX	Jan 1989	May 1989
Venezuela	VEN	Feb 1990	Dec 1990
India	IND	Dec 1992	Nov 1992
Korea	KOR	Feb 1992	Jan 1992
Malaysia	MAL	Jan 1989	- -
Pakistan	PAK	Apr 1991	Feb 1991
Phillipines	PHI	Jan 1989	Nov 1991
Taiwan	TAI	Feb 1991	Jan 1991
Thailand	THA	Jan 1989	- -
Greece	GRE	Jan 1989	- -
Jordan	JOR	Jan 1989	- -
Nigeria	NIG	Not available	- -
Zimbabwe	ZIM	Nov 1993	- -

PART II

Modeling Risk Premia in Futures Markets

Chapter 5

An Introduction to Modeling Risk Premia in Futures Markets

5.1 Introduction

In this chapter we present a short introduction to the literature on modeling risk premia in futures markets. In Chapter 3 it was shown that, in spanning and intersection tests, the main characteristic of a futures contract is that it is a zero-investment security. In the next section we will show that this zero-investment property also implies that expected futures returns consist of risk premia only and that they do not contain a risk free rate or zero beta return component, as is the case with non-zero investment securities like stocks and bonds. Knowledge about futures risk premia is important, because the futures risk premia affect for instance the portfolio decisions of investors and the hedge decisions of companies. In the last section of this chapter it is explained in detail how risk premia affect hedging decisions. The effect of futures risk premia on portfolio decisions is easily understood within the spanning and intersection framework discussed in Part I of this thesis. Because in the literature on futures markets the risk premia that investors expect to earn in period $t+1$ are often related to variables that can be observed at time t, models for futures risk premia suggest which variables can be used as conditioning variables in the spanning and intersection regressions of Part I. For instance, especially in the commodity futures literature, futures returns are related to so-called hedging pressure variables, which we also used in Chapter 3. These hedging pressure variables measure the aggregate nonmarketable risks faced by investors, such as the exposure to currency risk faced by importing and exporting firms, the inflation risk faced by pension funds and the commodity price risk faced by many farmers and industrial companies. It is well known by now that futures returns can be predicted from their own hedging pressure (see, e.g., Carter, Rausser & Schmitz (1983), Chang (1985) and Bessembinder (1992)). In Chapter 6 we analyze a model in which futures risk premia do not only depend on their own hedging pressure, but also on the hedging pressure from other futures markets.

It is also well known that futures risk premia depend on the current forward spread, i.e., the difference between the current futures price and the spot price of the asset underlying the futures contract. This variable is used in the empirical analyses of many futures markets. For instance, Fama (1984a), McCurdy & Morgan (1987) and Peresetsky & DeRoon (1997) use this variable in analyzing currency futures, Fama (1984b, 1984c) uses this variable in analyzing interest rate futures and Fama & French (1987) use it in analyzing commodity futures. In Chapter 7 we will use the spreads between futures prices in a similar way to analyze differences in risk premia for futures contracts that only differ in their maturity.

The remainder of this chapter is organized as follows. In the next section we will give a brief introduction to modeling futures risk premia. This introduction serves as the basis of the analysis in Chapter 6. In Section 5.3 we extend this introduction to the analysis of risk premia for futures contracts with different maturities, a subject that will be discussed in detail in Chapter 7. Finally, in the last section it is shown how the analysis of futures risk premia relates to tests for intersection and spanning and how knowledge of futures risk premia can be used in hedge decisions.

5.2 Modeling expected futures returns

It was already indicated in Chapter 3 that an important difference between futures and securities like stocks and bonds stems from the fact that futures contracts (like forward contracts and swaps for instance) do not require an initial investment. Whereas a stock or a bond requires an investment of P_t at time t and yields a payoff P_{t+1} (including dividends, coupon payments and the like) at time $t+1$, a futures contract requires an initial investment of 0 and yields a payoff $F_{t+1} - F_t$ at time $t+1$, i.e., the difference between the futures price at time $t+1$ and t respectively. The futures price F_t is the price at time t for delivery of the asset underlying the futures contract at some fixed date in the future. Assume that an investor has an amount Y_t that he can invest in assets like stocks and bonds. Besides these investments he can also add futures and forward contracts to his portfolio without any additional capital requirements. Thus, we assume that there are no margin requirements or that the investor can use stocks or bonds to fulfill his margin requirements. Denote the K-dimensional vector of portfolio weights in non-zero investment securities like stocks and bonds as w_A and the L-dimensional vector of portfolio weights in futures contracts as w_F. The elements of w_A and w_F are both expressed as fractions of Y_t. Thus, if the investor buys N_i securities i, then $w_i = N_i P_{i,t}/Y_t$ if security i is a non-zero investment security and $w_i = N_i F_{i,t}/Y_t$ if security i is a zero investment security like a futures contract. Because the investor has to invest an amount Y_t he faces the restriction that $w'_A \iota_K = 1$, whereas there are no restrictions on w_F. His portfolio return is therefore equal to

$$r^p_{t+1} = w'_A r_{A,t+1} + w'_F r_{F,t+1}, \tag{5.1}$$

where $r^p_{t+1} = Y_{t+1}/Y_t$, $r_{A,t+1} = P_{t+1}/P_t$ and $r_{F,t+1} = (F_{t+1} - F_t)/F_t$. Notice the difference between the return definitions for futures and non-zero investment securities that arises because returns are defined as payoffs scaled by price. Also note that the term futures return is actually a misnomer because of the zero investment nature of the contract.

As in Section 2.1, consider the one-period problem where the investor maximizes his indirect utility of wealth function

$$\begin{aligned} \max_{\{w_A, w_F\}} & \; E_t[u(Y_t r^p_{t+1})], \\ \text{s.t. } r^p_{t+1} &= w'_A r_{A,t+1} + w'_F r_{F,t+1}, \\ w'_A \iota_K &= 1. \end{aligned}$$

Solving this problem and defining the pricing kernel $M_{t+1} = Y_t u'(Y_t r^p_{t+1})/\eta$, with η the Lagrange multiplier for the restriction that $w'_A \iota_K = 1$, gives the first order conditions

$$E_t[M_{t+1} r_{A,t+1}] = \iota_K, \tag{5.2a}$$

$$E_t[M_{t+1} r_{F,t+1}] = 0. \tag{5.2b}$$

Denoting the expectation of the pricing kernel as $E_t[M_{t+1}] = v_t$ and using the definition of covariance, this gives for the expected returns

$$E_t[r_{A,t+1}] = \frac{1}{v_t} - \frac{Cov_t[r_{A,t+1}, M_{t+1}]}{v_t}, \tag{5.3a}$$

$$E_t[r_{F,t+1}] = -\frac{Cov_t[r_{F,t+1}, M_{t+1}]}{v_t}. \tag{5.3b}$$

The important thing to note about these relations is that, whereas the expected returns on non-zero investment securities are determined by $1/v_t$ plus a risk premium which is determined by the covariance of the asset return with the pricing kernel, expected futures returns are determined by a risk premium only. Note that $1/v_t$ equals the risk free rate if a risk free asset exists.

From (5.3b) it follows immediately that knowledge about the futures risk premium is equivalent to knowledge about the expected futures return. The second part of this thesis focuses on this risk premium, i.e., on the covariance of the pricing kernel with the futures return. In the next chapter we will analyze the functions that several asset pricing models assign to the pricing kernel. As noted in Section 2.2.1, different asset pricing models assign different functions to M_{t+1}. For instance, the (unconditional) CAPM implies that the kernel is of the form $a + br^m_{t+1}$, where r^m_{t+1} is the return on the market portfolio. Leaving out the time subscripts in (5.3) and noting that M_{t+1} should price

r^m_{t+1} correctly, (5.3a) implies

$$E[r^m_{t+1}] = \frac{1}{v} - \frac{b}{v} Var[r^m_{t+1}] \Leftrightarrow \frac{b}{v} = -\frac{E[r^m_{t+1}] - \frac{1}{v}}{Var[r^m_{t+1}]}.$$

Combining this with (5.3b) and substituting $a + br^m_{t+1}$ for M_{t+1} there as well, we have for expected futures returns:

$$E[r_{F,t+1}] = -\frac{b}{v} Cov[r_{F,t+1}, r^m_{t+1}] = \beta_F E[r^m_{t+1} - \frac{1}{v}], \tag{5.4}$$

with $\beta_F = Cov[r_{F,t+1}, r^m_{t+1}]/Var[r^m_{t+1}]$. Equation (5.4) gives the familiar beta-form of the CAPM for expected futures returns. The validity of the CAPM as a description of expected futures returns has been analyzed by, e.g., Dusak (1973) and Black (1976).

Other specifications of the pricing kernel in relation to futures pricing have been suggested for instance by Jagannathan (1985), who uses the Consumption based CAPM, and Stoll (1979) and Hirshleifer (1988a, 1989), who analyze models in which investors face nonmarketable risks and use futures markets to hedge these risks. Along the same lines, in the next chapter we will derive a multifactor model in which futures risk premia depend on the covariance of the futures return with the return on the market portfolio, as well as the nonmarketable risks that investors want to hedge. Because all these models are derived under rather restrictive assumptions, the implied pricing kernel is only a *proxy* stochastic discount factor and will in general not price the securities in the dataset correctly. In the next chapter we analyze the amount of misspecification in the proposed model for a set of 20 (financial and commodity) futures contracts. To this end we will employ the specification error bound introduced by Hansen & Jagannathan (1997) which was discussed in Section 6 of Chapter 2.

5.3 Expected returns for futures with different maturities

A distinctive feature of futures markets is that at each point in time t several futures contracts are traded that have the same underlying value but that differ with respect to the delivery date, $t+n$. The futures price at time t for delivery at time $t+n$ is denoted as $F^{(n)}_t$. By a no arbitrage argument, the relation between the futures price $F^{(n)}_t$ and the current spot price S_t of the underlying asset[16] is given by the *cost-of-carry* model:

$$F^{(n)}_t = S_t \exp\{ny^{(n)}_t\}, \tag{5.5}$$

16 We denote the price of an asset underlying a futures contract by S_t to distinguish it from the price of financial assets like stocks or bonds, which are denoted by P_t, and to stress the fact that the asset underlying a futures contract does not have to be a traded asset. This will be discussed in more detail in the next section.

where $y_t^{(n)}$ is the continuously compounded cost-of-carry for holding the asset from time t until time $t+n$. In general, the cost-of-carry consists of the n-period interest rate plus the cost of holding (storing) the asset from time t to $t+n$, minus the benefits that the owner of the asset receives from holding the asset from t to $t+n$, and which consist of elements like dividends and convenience yields (see, e.g., Hull (1997)). This cost-of-carry will also be referred to as the *yield*. Because the yield $y_t^{(n)}$ may be different for different t as well as different n, expected returns (and therefore risk premia) on futures contracts that only differ in their remaining maturity may differ as well. In analyzing risk premia on futures contracts with different maturities, a topic that will be discussed in detail in Chapter 7, it is useful to use a *log*-approximation to one-period futures returns:

$$r_{f,t+1}^{(n)} \equiv f_{t+1}^{(n-1)} - f_t^{(n)} \approx \frac{F_{t+1}^{(n-1)} - F_t^{(n)}}{F_t^{(n)}} \equiv r_{F,t+1}^{(n)},$$

with $f_t^{(n)} \equiv \log F_t^{(n)}$. Use of the cost-of-carry relation in (5.5) gives for the one-period futures return with initial maturity n:

$$\begin{aligned} r_{f,t+1}^{(n)} &= f_{t+1}^{(n-1)} - f_t^{(n)} = (s_{t+1} - s_t) + ((n-1)y_{t+1}^{(n-1)} - ny_t^{(n)}) \\ &= r_{s,t+1} + ((n-1)y_{t+1}^{(n-1)} - ny_t^{(n)}). \end{aligned} \tag{5.6}$$

Thus, futures returns can be decomposed in two parts: the return on the underlying asset, $r_{s,t+1}$, plus the change in the yield. Notice that even if there is a flat and deterministic term structure of yields, i.e., $y_t^{(n)} = y, \forall t, n$, futures returns differ from spot returns by exactly y. Also note that the difference between returns on futures contracts with different maturities can be expressed in terms of differences in yields. Differences in risk premia for futures contracts with different maturities can therefore also be attributed to differences in yields. Using (5.6), the risk premium on futures contracts can be written as[17]

$$\begin{aligned} -\frac{Cov_t[r_{f,t+1}, M_{t+1}]}{v_t} &= -\frac{Cov_t[r_{s,t+1}, M_{t+1}]}{v_t} - (n-1)\frac{Cov_t[y_{t+1}^{(n-1)}, M_{t+1}]}{v} \\ &= -\pi_{s,t} - \pi_t^{(n)}, \end{aligned} \tag{5.7}$$

with $\pi_{s,t} \equiv Cov_t[r_{s,t+1}, M_{t+1}]/v_t$ and $\pi_t^{(n)} \equiv (n-1)Cov_t[y_{t+1}^{(n-1)}, M_{t+1}]/v_t$. In Chapter 7 we will analyze whether the current term structure of futures prices contains information about the term premium $\pi_t^{(n)}$ and derive a simple model for $\pi_t^{(n)}$ using a one-factor model for $y_t^{(n)}$.

The fact that $y_t^{(n)}$ is a stochastic variable causes futures contracts and the underlying value to be less than perfectly correlated. Therefore, as we will see in the next section, in general it will not be possible to create a perfect hedge with futures contracts (i.e., a hedge that results in a risk

[17] Given that we use *log* returns, this is again an approximation.

free position) unless the maturity of the hedge coincides with the maturity of an available futures contract. If this is not the case, then the hedge will be less than perfect because of uncertainty in $y_t^{(n)}$, which is known as *basis risk*. In general, basis risk arises because at the end of the hedge period the futures price differs from the price of the asset to be hedged. This may happen because of uncertainty in the yield, as mentioned, or because the asset underlying the futures contract is not (exactly) the same asset as the asset to be hedged, a situation which is known as a *cross hedge*.

5.4 Hedging with futures contracts

Futures contracts provide a valuable tool in hedging positions, for both financial and non-financial firms. From Equation (5.6) it follows that the futures return can be decomposed in the return on the underlying value plus the change in the yield. Because the variability in the yields is usually much smaller than the variability in the underlying value, the returns on the futures contract and the underlying asset (portfolio) will be highly correlated. This correlation will be lower when there is more basis risk. Given that in general the futures price moves closely in line with the asset price, asset price movements can be neutralized, or hedged, by taking an opposite position in futures contracts. In this section we will analyze the use of futures contracts for hedging purposes. It was shown in Chapter 3 how the spanning and intersection tests described in Part I of this thesis can be used to test whether investors can improve the efficiency of their portfolio by adding futures contracts to their portfolio. There it was also shown how spanning and intersection tests can be adjusted to account for nonmarketable risks that agents want to hedge. Let q be a S-dimensional vector containing the size of nonmarketable positions faced by the investor. As in Chapter 3, q_s is expressed as a fraction of the wealth Y_t invested in the K assets, implying that the total wealth of the investor is equal to $(1 + q'\iota_S)Y_t$, which may be smaller or larger than Y_t. The returns on the nonmarketable positions are denoted by the vector $r_{S,t+1}$, implying that the total return on the investors portfolio is equal to

$$r_{t+1}^p = w_A' r_{A,t+1} + w_F' r_{F,t+1} + q' r_{S,t+1}. \tag{5.8}$$

For the commodities and currencies in Chapter 3, we tested whether an investor that initially invests in the stock indices of the US, the UK and Germany can benefit from hedging with futures if he has a nonmarketable position of 25% (i.e., $q_s = 0.25$) in a commodity or currency. For nonmarketable positions in commodities, for investors with mean-variance utility functions as well as investors with log utility or power utility functions, the hypothesis that they can not benefit from hedging with futures contracts is rejected in Chapter 3. For nonmarketable positions in currencies this is only the case for investors with a power utility function. Similarly, Glen & Jorion (1993) analyze whether investors with a mean-variance utility function can improve the efficiency of international portfolios of stocks and bonds by hedging the currency risk associated with the portfolio. They find that adding currency futures causes a (economically and statistically) significant increase in the Sharpe ratio for international portfolios containing bonds, but not for international portfolios that consist of stocks only. When they use the forward premium (i.e., $F_t - S_t$) as a conditional variable however, there is a significant increase in the Sharpe ratio for all cases considered.

As outlined in Chapter 3, nonmarketable positions can be incorporated in tests for spanning and intersection by using exposure adjusted returns for the non-zero investment securities, i.e., $\widetilde{r}_{A,t+1} = r_{A,t+1} + q' r_{S,t+1} \iota_K$, whereas the returns on the futures contracts are unadjusted. For an investor who has a number of nonmarketable positions that are given by the vector q, and who has a portfolio of stocks and bonds, the question whether or not he should hedge his exposure with futures contracts can be answered using a test for mean-variance intersection, based on the regression

$$r_{F,t+1} = \alpha + \beta \widetilde{r}_{A,t+1} + \varepsilon_{t+1}. \tag{5.9}$$

Given a zero-beta rate η, testing for intersection means testing the restrictions $\alpha + \eta \beta \iota_K = 0$. If the investor considers to add futures contracts to his portfolio, the estimated parameters in (5.9) can be used to obtain consistent point estimates of the optimal futures positions as well as the new optimal portfolio weights for the assets $r_{A,t+1}$.

Models for futures risk premia often suggest that the futures risk premia depend on variables that can be observed at time t. For instance, the model that will be derived in Chapter 6 suggests that futures risk premia depend on the market hedging pressures, i.e., the aggregate nonmarketable positions q of all investors at time t. Therefore, these market hedging pressures can be used as conditioning variables in the intersection regression in (5.9). Similarly, in Chapter 7 information about the term premiums $\pi_t^{(n)}$ is derived from the spread between 1-period and n-period log futures prices, i.e., from $f_t^{(n)} - f_t^{(1)}$. Assuming that only expected returns depend on the conditioning variables z_t, whereas all variances and covariances are constant, the conditioning variables can be incorporated in the intersection regression (5.9) in the way described in Section 4.2, by including z_t as additional regressors:

$$r_{F,t+1} = \alpha_0 + \alpha_1 z_t + \beta \widetilde{r}_{A,t+1} + \varepsilon_{t+1}. \tag{5.10}$$

Assuming that a risk free asset with return R^f is available, a straightforward generalization of (2.62) shows that the optimal futures positions conditional on z_t and an expected portfolio return equal to m are given by

$$\begin{aligned} w_F^*(z_t) &= \frac{m - R^f}{\theta^2} \Sigma_{\varepsilon\varepsilon}^{-1} \alpha_J(z_t), \quad (5.11) \\ \text{with } \alpha_J(z_t) &= (\alpha_0 + \alpha_1 z_t) + R^f \beta \iota. \end{aligned}$$

The new optimal portfolio weights for the assets that were already in the portfolio can in a similar way be derived from (2.63). These results motivate the usefulness of models for futures risk premia, because these models often suggest which variables can be used as conditioning variables in hedge and/or portfolio decisions.

Perhaps the role of the futures risk premia can be seen even more clearly when we solve the mean-variance portfolio problem when there are futures contracts and nonmarketable risks. If the portfolio return is given by (5.8) and if it is assumed again that only expected returns are time-varying, whereas all (co)variances are constant, the investor solves the problem

$$\begin{aligned} & \max_{w_A, w_F} f(E_t[r_{t+1}^p], Var[r_{t+1}^p]), \\ \text{s.t. } E_t[r_{t+1}^p] &= w_A' E_t[r_{A,t+1}] + w_F' E_t[r_{F,t+1}] + q' E_t[r_{S,t+1}], \\ Var[r_{t+1}^p] &= w' \Sigma_{rr} w + 2 w' \Sigma_{rS} q + q' \Sigma_{SS} q, \\ w_A' \iota_K &= 1, \end{aligned}$$

where $w = (w_A' \ w_F')'$, $r_{t+1} = (r_{A,t+1}' \ r_{F,t+1}')'$, $\Sigma_{rr} = Var[r_{t+1}]$, $\Sigma_{rS} = Cov[r_{t+1}, r_{S,t+1}]$ and $\Sigma_{SS} = Var[r_{S,t+1}]$. From the first order conditions of this problem it is straightforward to derive the optimal portfolio weights:

$$\begin{aligned} \begin{pmatrix} w_A \\ w_F \end{pmatrix}^* &= \gamma^{-1} \begin{pmatrix} \Sigma_{AA} & \Sigma_{AF} \\ \Sigma_{FA} & \Sigma_{FF} \end{pmatrix}^{-1} \begin{pmatrix} E_t[r_{A,t+1}] - \eta \iota_K \\ E_t[r_{F,t+1}] \end{pmatrix} \quad (5.12) \\ & - \begin{pmatrix} \Sigma_{AA} & \Sigma_{AF} \\ \Sigma_{FA} & \Sigma_{FF} \end{pmatrix}^{-1} \begin{pmatrix} \Sigma_{AS} \\ \Sigma_{FS} \end{pmatrix} q, \end{aligned}$$

where $\gamma = -\frac{1}{2} f_2(.)/f_1(.)$, the risk aversion parameter, and the covariance matrices are in obvious notation. If there are only futures contracts available and no non-zero investment securities, Equation (5.12) specializes to a well-known result in Anderson & Danthine (1981):

$$w_F^* = \gamma^{-1} \Sigma_{FF}^{-1} E_t[r_{F,t+1}] - \Sigma_{FF}^{-1} \Sigma_{FS} q. \quad (5.13)$$

Equations (5.12) and (5.13) show that the optimal portfolio consists of two parts. The first part is the traditional mean-variance efficient portfolio if there are no nonmarketable risks, and is known as the *speculative demand* for assets and futures. The second part depends on the size of the nonmarketable risks, and is known as the *hedge demand,* which can be written in short as $-\Sigma_{rr}^{-1} \Sigma_{rs} q$.

If the investor is very risk averse, i.e., $\gamma \rightarrow \infty$, the speculative demand tends to zero and the optimal portfolio is determined by the hedge demand only. This result is also obtained if all expected futures returns and excess asset returns are equal to zero. Notice that the hedge demand positions can be estimated from a least-squares regression of the nonmarketable risks on the asset and futures returns:

$$r_{S,t+1} = \alpha + \beta_A' r_{A,t+1} + \beta_F' r_{F,t+1} + \varepsilon_{t+1}, \tag{5.14}$$

with $E[\varepsilon_{t+1}] = E[\varepsilon_{t+1} r_{t+1}] = 0$. It is easy to see that the OLS-estimate of $\beta = (\beta_A' \ \beta_F')'$ gives a consistent estimate of $\Sigma_{rr}^{-1}\Sigma_{rS}$. Given the OLS-estimate $\widehat{\beta}$, the estimated hedge demand is therefore equal to $-\widehat{\beta}q$. Notice that if $q = 1$, the variance of an unhedged position is $Var[r_{S,t+1}]$ whereas the variance of a position hedged with β_A assets and β_F futures has a variance of $Var[\varepsilon_{t+1}]$. Because the OLS-regression minimizes the variance of the residual, it gives hedge ratios $\widehat{\beta}_A$ and $\widehat{\beta}_F$ that minimize the variance of the residuals, i.e., the variance of a hedged position. Therefore the OLS hedge ratios $\widehat{\beta}_A$ and $\widehat{\beta}_F$ are also known as the minimum variance hedge ratios. Since the purpose of hedging is to reduce the variance of the portfolio return, it is natural to use the decrease in the variance as a measure of the effectiveness of the hedge. In case of the minimum variance hedge, the R^2 of the regression in (5.14) is therefore a natural measure of the hedge effectiveness. DeJong, DeRoon & Veld (1997) analyze the hedging effectiveness of currency futures for the minimum variance hedge as well as for a number of other hedges.

If there is only one nonmarketable risk and only one futures contract available, the hedge demand is equal to

$$w_F^* = -\frac{Cov[r_{F,t+1}, r_{S,t+1}]}{Var[r_{F,t+1}]} q.$$

Using the log approximation from the previous section, and assuming that the nonmarketable risk is the underlying value of the futures contract, the numerator of the hedge demand is

$$Cov[r_{s,t+1} + (n-1)y_{t+1}^{(n-1)}, r_{s,t+1}] = Var[r_{s,t+1}] + (n-1)Cov[y_{t+1}^{(n-1)}, r_{s,t+1}].$$

The latter term in this expression will in general not be equal to zero, unless $n = 1$ or $y_{t+1}^{(n-1)}$ is constant. The fact that the covariance term is not equal to zero, reflects the presence of basis risk. If this basis risk is absent, the spot and the futures price are perfectly correlated and $w_F^* = -q$. In this case a perfect hedge is possible, meaning that the nonmarketable risk faced by the investor can be perfectly offset by taking a position in the futures market that has equal size but opposite sign as the nonmarketable position.

From (5.12) and (5.13) it follows that the futures risk premia only affect the speculative demand. Depending on their risk aversion, investors will deviate from the pure hedge positions in order to benefit from the risk premia that can be earned in futures markets (and other markets as well). Notice that the hedge demand of the investor does not depend on the expected returns on the available securities, whereas the speculative demand does not depend on the nonmarketable risks faced by the investor. However, in the next chapter we will show that if all investors choose their portfolio according to (5.12) and markets clear, expected returns are linear functions of the aggregate nonmarketable risks faced by investors. Thus, although the speculative demand is not directly influenced by the individual investors' nonmarketable risks, it does depend on the aggregate nonmarketable risks of all investors.

Chapter 6

Specification Error Analysis in Models for Futures Risk Premia with Hedging Pressure

6.1 Introduction

As outlined in the previous chapter, futures prices are known to deviate from expected future spot prices because of risk premia that traders expect to earn (or pay) when trading in futures markets. Futures risk premia are important, because they affect the costs and benefits of hedging as well as the diversification benefits that result from including futures in investment portfolios. Also, to the extent that economic agents make their production, storage, and consumption decisions by looking at the pattern of futures prices as an indicator of future spot prices, it is important to know the bias that exists in futures prices. There is an ongoing debate about the determinants of futures risk premia. Futures risk premia are usually related to systematic risk, as in the work of Dusak (1973), Black (1976), and Jagannathan (1985) e.g., and to net positions of hedgers in futures markets, which is known as hedging pressure. The use of hedging pressure as an explanation for the futures price bias dates back to Keynes and Hicks, and has more recently been incorporated in models that allow both hedging pressure and systematic risk to affect futures prices (see, e.g., Stoll (1979) and Hirshleifer (1988a, 1989)). In these models, futures risk premia are typically determined by the covariance of the futures returns with the market return and by the futures own hedging pressure. Carter, Rausser & Schmitz (1983) and Bessembinder (1992) provide empirical evidence for this combined role of futures contracts own hedging pressures and systematic risk as measured by the covariance between the futures returns and the market return or other economic aggregates.

In this chapter we present a simple model in the spirit of Stoll (1979) and Hirshleifer (1988a, 1989) in which agents face multiple sources of nonmarketable risks. In equilibrium this implies a multifactor model as discussed for instance in Fama (1996) in which the market portfolio is multifactor efficient and where the nonmarketable risks are the relevant state variables. Using hedging pressure variables as proxies of the aggregate net positions in the nonmarketable risks of all the agents trading in financial markets, the model implies that futures returns are determined by the covariance of the futures return with the market return, as well as by hedging pressure, as is the case in previous models. The distinguishing feature of the model presented in this chapter is that the futures risk premium is not only determined by its own hedging pressure, but also by hedging pressures from other markets, i.e., by cross hedging pressure effects. A second contribution of this chapter is that we use the measure for misspecification introduced by Hansen & Jagannathan (1997) to analyze the specification errors in the model. This measure gives an indication of the extent to which (portfolios of) futures contracts are mispriced by the model.

We analyze the effect of both market risk and hedging pressure variables on futures risk premia for 20 futures markets that are divided into four groups: financial futures, agricultural futures, mineral futures, and currency futures. The dataset consists of semimonthly observations for the period January 1986 until December 1994. For these markets we find that both the futures own hedging pressure as well as cross hedging pressure variables from within the futures own group are important in explaining futures returns. We also find that a multifactor model with hedging pressure variables from within the futures own group produces specification error bounds that are substantially smaller than those produced by a model without hedging pressure, although the specification error bounds are quite large for both models.

The remainder of this chapter is organized as follows. In the next section we will first present a multifactor model for futures returns. In Section 6.3 we will discuss the specification errors introduced by Hansen & Jagannathan (1997). Section 6.4 describes the data, and in Sections 6.5 and 6.6 we will provide an empirical analysis of the multifactor model for 20 futures contracts. The chapter ends with some concluding remarks.

6.2 Modeling futures risk premia

There is an extensive literature[18], both theoretical and empirical, that relates futures risk premia to two determinants: systematic risk and hedging pressure. In mean-variance models, if all risks are perfectly marketable or if all agents have free access to the available financial markets, then agents can freely diversify their portfolios and futures risk premia depend on systematic risk only, i.e., on the covariation between futures returns and the market return. An essential ingredient in models using hedging pressure as a determinant of futures price movements, is that a subgroup of agents face nonmarketable risks and that some of these agents do not have free access to futures or other financial markets in order to hedge these risks. This limited access to financial markets may be caused by market frictions such as informational barriers or transaction costs, as in Hirshleifer (1988a). To show the nature of models containing both systematic risk and hedging pressure, we use the economic setting that was introduced in the previous chapter. Thus, there are K assets available in which agents can invest, as well as L futures markets. The net returns on the K assets

[18] E.g., Stoll (1979) , Hirshleifer (1988a, 1989), Carter, Rausser & Schmitz (1983) and Bessembinder (1992).

are denoted by the L-dimensional vector $r_{A,t+1}$, whereas the returns on the L futures contracts are denoted by $r_{F,t+1}$.[19] Apart from these marketable securities, the end of period wealth of an agent may be affected by S nonmarketable positions, the returns on which are given by the S-dimensional vector $r_{S,t+1}$. It is assumed that the portfolio problem can be described in terms of mean and variance of the portfolio return only. As in the previous chapter, this portfolio return is given by

$$r^p_{t+1} = w'_A r_{A,t+1} + w'_F r_{F,t+1} + q^{j\prime} r_{S,t+1}, \tag{6.1}$$

where w_A is the vector of portfolio weights in the K assets, w_F the vector of positions in the L futures contracts, and q^j the sizes of the S nonmarketable positions faced by agent j, expressed as a fraction of wealth invested in financial markets. Although we will explicitly allow for time variation in the nonmarketable positions later on, at this stage we leave out time subscripts for w_A, w_F, and q^j in order to keep notation simple. Throughout the analysis we will make the assumption that q^j_s is known at the beginning of the period. If $r_{s,t+1}$ refers to the return on nonmarketable commodities for instance, this assumption implies that we assume there is no quantity risk[20]. The weights w_A, as well as w_F and q^j, are all fractions of the amount invested in the K assets, implying that the asset weights in w_A sum to one. It is common practice in the futures market literature to distinguish agents according to a nonmarketable position q^j_s as producers ($q^j_s > 0$), consumers ($q^j_s < 0$), or speculators ($q^j_s = 0$). Hirshleifer (1988b) makes a further distinction in primary suppliers (growers) and intermediate processors of commodities. Notice that because here we explicitly allow for multiple nonmarketable positions, agent j may be a producer with respect to one nonmarketable asset and a consumer or speculator with respect to others.

Define the $(K{+}L)$-dimensional vectors $w \equiv (w'_{A,t}\ w'_{F,t})'$ and $r_{t+1} \equiv (r'_{A,t+1}\ r'_{F,t+1})'$. Given the assumption made earlier that the portfolio problem of the agent depends on the mean and variance of portfolio return only, the problem that agent j has to solve is, using obvious notation,

$$\max_{\{w\}} f^j(E[r^p_{t+1}], Var[r^p_{t+1}]), \tag{6.2a}$$

$$\begin{aligned}
\text{with } E[r^p_{t+1}] &= w'E[r_{t+1}] + q^{j\prime}E[r_{S,t+1}], && (6.2b)\\
Var[r^p_{t+1}] &= w'Var[r_{t+1}]w + 2w'Cov[r_{t+1}, r_{S,t+1}]q^j && (6.2c)\\
&\quad + q^{j\prime}Var[r_{S,t+1}]q^j, &&\\
\text{and s.t. } w'_A\iota &= 1, && (6.2d)
\end{aligned}$$

[19] Notice that because of the zero-investment nature of futures contracts, the term futures return is actually a misnomer.
[20] This is not very restrictive within the framework considered here however, since we can always adjust the definition of $r_{S,t+1}$ to allow for quantity risk.

where f^j is increasing in its first argument and decreasing in its second argument, and where ι is a K-dimensional vector of ones. Differentiating with respect to w, the first order conditions imply for the expected asset and futures returns respectively:

$$E[r_{A,t+1}] - \eta\iota = \gamma^j\{Cov[r_{A,t+1}, r_{t+1}]w^* + Cov[r_{A,t+1}, r_{S,t+1}]q^j\}, \quad (6.3a)$$

$$E[r_{F,t+1}] = \gamma^j\{Cov[r_{F,t+1}, r_{t+1}]w^* + Cov[r_{F,t+1}, r_{S,t+1}]q^j\}, \quad (6.3b)$$

where $\gamma^j = -\frac{1}{2}f_2^j(.)/f_1^j(.)$ and η is the Lagrange-multiplier for the restriction that $w_A'\iota = 1$, which equals the zero-beta return that corresponds to the optimal portfolio w^*. If there exists a risk free asset, one can easily show that the same result holds with $\eta = r^f$. The optimal asset and futures positions w^* are easily derived from (6.3) and lead to similar formulas as in Anderson & Danthine (1981).

An equilibrium model can be obtained if it is assumed that all agents that enter the financial markets solve the problem in (6.2) and that markets clear. In that case the market portfolio w^m is multifactor efficient (ME) and satisfies (6.3). Notice that since futures contracts are in zero net supply, the market portfolio is of the form $w^m = (w_A^{*\prime}\ 0')'$, i.e., futures contracts do not enter the market portfolio. Fama (1996) presents formulas that are similar to (6.3) in his derivation of Merton's Intertemporal CAPM (the ICAPM). Indeed, the expressions in (6.3) readily lead to a version of the ICAPM in which the relevant state variables are the returns on the S nonmarketable positions $r_{S,t+1}$. The target loadings on the S state variables in Fama (1996) are in this case entirely determined by the size of the nonmarketable positions q^j. Portfolios w^* that satisfy (6.3) for some choice of γ and q are therefore called Multifactor-Minimum Variance (MMV) portfolios rather than Minimum-Variance (MV) portfolios.

If the wealth of agent j invested in assets is denoted by Y_t^j, the aggregate nonmarketable position $q_{s,t}^m$ is given by

$$q_{s,t}^m = \frac{\sum_{j=1}^N Y_t^j q_{s,t}^j}{\sum_{j=1}^N Y_t^j}, \quad (6.4)$$

where we now explicitly allow for time variation in $q_{s,t}^j$. For simplicity it is assumed that variances and covariances do not vary over time. In Appendix 6.A it is shown that if the market portfolio is indeed ME, the expected asset and futures returns satisfy:

$$E_t[r_{A,t+1}] - \eta\iota = \beta_A E_t[r_{t+1}^m - \eta] + \sum_{s=1}^S \theta_{A,s} q_{s,t}^m, \quad (6.5a)$$

$$E_t[r_{F,t+1}] = \beta_F E_t[r_{t+1}^m - \eta] + \sum_{s=1}^S \theta_{F,s} q_{s,t}^m, \quad (6.5b)$$

where β_i has the familiar beta-interpretation, i.e., $\beta_i = Cov[r_{i,t+1}, r^m_{t+1}]/Var[r^m_{t+1}]$, and where $\theta_{i,s}$ is given by

$$\theta_{i,s} = \gamma^m\{Cov[r_{i,t+1}, r_{s,t+1}] - \beta_i Cov[r^m_{t+1}, r_{s,t+1}]\}, \tag{6.6}$$

with γ^m the market risk aversion parameter.

If q^m_t is time-varying, the model in (6.5a) and (6.5b) implies that expected returns are also time-varying. In case of futures markets, the nonmarketable positions $q_{s,t}$ are usually associated with (planned) future production and consumption of commodities. In that case, the total risk faced by all producers has the same size, but opposite sign, of the total risk faced by all consumers. When all producers and consumers enter the financial markets, $q^m_{s,t}$ equals zero, since in that case the nonmarketable risks of producers and consumers would cancel out (see also Hirshleifer, (1990)). However, if market frictions such as transaction costs or informational barriers deter some traders from participating in financial markets[21], $q^m_{s,t}$ will in general not be equal to zero and will affect the risk premiums on both assets and futures contracts as can be seen from (6.5a) and (6.5b), (see also Stoll (1979) and Hirshleifer (1988a, 1989)). The result that aggregate positions in nonmarketable risks affect the expected returns in futures and asset markets is the well known hedging pressure effect.

Summarizing, the model for futures risk premia in (6.5b) shows that futures risk premia are determined by two kinds of variables: systematic risk as measured by the futures beta with respect to the market portfolio, and hedging pressure, as measured by $\theta_{i,s}$, the sensitivity of futures contract i with respect to nonmarketable position s. In empirical work (see, e.g., Carter, Rausser & Schmitz (1983) and Bessembinder (1992)), futures risk premia are usually related to market risk and the futures own hedging pressure. According to (6.5b) however, the futures risk premium is not only determined by its own hedging pressure, but also by other hedging pressures, i.e., by *cross* hedging pressures. As noted by Anderson & Danthine (1981), cross hedging may arise because the cash and the futures price are not perfectly correlated (because of basis risk) or because agents may be concerned about hedging cash positions for which no futures contracts are traded. In the empirical section we will investigate the importance of these cross hedging pressure effects.

As a final point, notice that in studies on futures risk premia the interest is usually in a pricing model like (6.5b), where expected futures returns are related to hedging pressure variables, since it is generally believed that there are good proxies available for these variables. To the extent that these factors are unobserved however, it may be worthwhile to identify the corresponding factor mimicking portfolios (see, e.g., Fama (1996)).

21 Unlike Hirshleifer (1988a, e.g.), here we do not make a distinction between participation in stock markets and futures markets.

6.3 Specification error bounds in the multifactor model

The model derived in the previous section implies that risk premia for all futures contracts (as well as all assets) are determined by a systematic risk component as well as hedging pressure variables for all nonmarketable risks, reflecting all nonmarketable positions that agents may face. For futures, replacing expectations by realizations in (6.5b) gives

$$r_{i,t+1} = \alpha_i + \beta_i r^m_{t+1} + \sum_{s=1}^{S} \theta_{i,s} q^m_{s,t} + \varepsilon_{i,t+1}, \tag{6.7}$$

with $\alpha_i = -\beta_i \eta$,[22] $E[\varepsilon_{i,t+1}] = 0$ and $E[r^m_{t+1}\varepsilon_{i,t+1}] = E[q^m_{s,t}\varepsilon_{i,t+1}] = 0$, as can readily be checked. Given observations of futures returns, the market returns and the hedging pressure variables $q^m_{s,t}$, the model can be estimated and we can test the model by testing the restrictions for multifactor mean-variance efficiency of the market portfolio. Notice from (6.6) that $\theta_{i,s}$ in (6.7) depends on $Cov[r_{i,t+1}, r_{s,t+1}]$ and β_i, which will be close to zero in many cases. In the empirical analysis we will use this to restrict the number of hedging pressure variables that determine the futures risk premium. Obviously, a pricing model such as (6.5b) can only serve as an approximation. Thus, the stochastic discount factor implied by the model, which is of the form

$$y_{t+1} = a + b r^m_{t+1} + \sum_{s=1}^{S} c_s q^m_{s,t}, \tag{6.8}$$

will only yield approximate prices for the futures and asset returns, i.e.,

$$E[y_{t+1} r_{i,t+1}] = \pi^a(r_{i,t+1}). \tag{6.9}$$

Given that the Law of One Price holds, valid stochastic discount factors, denoted by m_{t+1}, will yield a price 1 to gross asset returns, i.e., $E[m_{t+1}(r_{a,t+1}+1)] = 1$, and a price 0 to futures returns, i.e., $E[m_{t+1} r_{f,t+1}] = 0$, reflecting the zero-investment property of futures contracts. Given that our specification of the stochastic discount factor will only yield approximate prices, we will analyze the extent to which the proxy stochastic discount factor y_{t+1} is misspecified by estimating the specification error bounds developed by Hansen & Jagannathan (1997), which were described in Section 2.6.

As in Chapter 2, let $\mathcal{M}$ be the set of admissable stochastic discount factors, i.e., discount factors that yield the correct prices to the securities under investigation. Recall that the measure of misspecification of a proxy stochastic discount factor y_{t+1} introduced by Hansen & Jagannathan

[22] If $r_{i,t+1}$ is the return on a nonzero-investment asset like a stock or a bond, the restriction is $\alpha_i = (1-\beta_i)\eta$.

(1997) is given by

$$\delta = \min_{m_{t+1} \in \mathcal{M}} \parallel y_{t+1} - m_{t+1} \parallel , \tag{6.10}$$

where $\parallel x_{t+1} \parallel \equiv E[x_{t+1}^2]^{\frac{1}{2}}$. Thus, δ is the minimum distance between the proxy stochastic factor y_{t+1} and the set of admissable stochastic discount factors. Also recall from Chapter 2 that Hansen & Jagannathan (1997) define an expected return error as

$$E^a[r_{f,t+1}] - E[r_{f,t+1}] = \frac{Cov[m_{t+1} - y_{t+1}, r_{f,t+1}]}{v},$$

for which

$$\mid E^a[r_{f,t+1}] - E[r_{f,t+1}] \mid \leq \frac{\sigma(y_{t+1} - m_{t+1})\sigma(r_{f,t+1})}{v},$$

as follows from the Cauchy-Schwarz inequality. Since this inequality holds for every valid stochastic discount factor m_{t+1} it must also hold for the stochastic discount factor that solves (6.10). Also, using the fact that y_{t+1} and m_{t+1} have the same expectation if y_{t+1} is associated with a linear factor model that includes a constant, it follows that

$$\mid E^a[r_{f,t+1}] - E[r_{f,t+1}] \mid \leq \frac{\delta\sigma(r_{f,t+1})}{v}, \tag{6.11}$$

Next note that the Sharpe ratio for a futures contract (or a portfolio of futures contracts) is simply the expected futures (portfolio) return divided by the standard deviation, i.e., $Sh(r_{f,t+1}) = E[r_{f,t+1}]/\sigma(r_{f,t+1})$. Similarly, the approximate Sharpe ratio implied by the proxy discount factor y_{t+1} is $Sh^a(r_{f,t+1}) = E^a[r_{f,t+1}]/\sigma(r_{f,t+1})$. Using these expressions, (6.11) readily yields

$$\mid Sh^a(r_{f,t+1}) - Sh(r_{f,t+1}) \mid \leq \frac{\delta}{v}. \tag{6.12}$$

Thus, the specification error bound, δ, scaled by the expectation of the stochastic discount factor gives an upper bound on the maximum absolute error in the Sharpe ratio of (portfolios of) the futures contracts under investigation. It is straightforward to include asset returns as well if the Sharpe ratios are based on the excess returns $r_{a,t+1} - 1/v$, i.e., asset returns in excess of $1/v$. This was also discussed in Section 2.6.

Estimation of the specification error bound for the proxy y_{t+1} implied by the models above gives an indication of how well the model performs in terms of implied Sharpe ratios (or expected returns). It is important to note though that although δ is the *minimum* distance between the proxy y_{t+1} and the set of admissable stochastic discount factors, it provides an *upper* bound on the errors in Sharpe ratios or expected returns. If the estimated specification error bounds are large, this should be interpreted with some care, since the most mispriced portfolios may imply taking extremely large positions in some securities. Hansen, Heaton & Luttmer (1995) show how restrictions on security positions can be taken into account when estimating the specification error bounds.

Details on consistent estimation of the specification error bound δ along with the unknown parameters a, b, and c_s, $s = 1, ..., S$, in (6.8) are given by Hansen & Jagannathan (1997). There it is shown that in case of a linear factor model as in (6.8) with unknown parameters a, b, and c_s, $s = 1, ..., S$, the problem in (6.10) can be written as:

$$\delta = \min_{\{a,b,c_s\}} \min_{\{m_{t+1} \in \mathcal{M}\}} \| a + br^m_{t+1} + \sum_{s=1}^{S} c_s q^m_{s,t} - m_{t+1} \| .$$

The solution to this problem is:

$$\delta = \min_{\{a,b,c_s\}} \| a\text{Proj}\{1 \mid r_{t+1}\} + b\text{Proj}\{r^m_{t+1} \mid r_{t+1}\} +$$

$$\sum_{s=1}^{S} c_s\text{Proj}\{q^m_{s,t} \mid r_{t+1}\} - \text{Proj}\{m_{t+1} \mid r_{t+1}\} \|,$$

where $\text{Proj}\{. \mid r_{t+1}\}$ denotes the least squares projection on the return vector r_{t+1} for the securities under investigation. Thus, estimating δ comes down to a simple least squares problem. Notice that $\text{Proj}\{1 \mid r_{t+1}\}$, $\text{Proj}\{r^m_{t+1} \mid r_{t+1}\}$,[23] and $\text{Proj}\{q^m_{s,t} \mid r_{t+1}\}$ are the factor mimicking payoffs. Given that the global minimum variance portfolio of r_{t+1} has positive systematic risk, these factor mimicking payoffs can be converted into factor mimicking *portfolios*, in which case the model in (6.5b) implies that the mean variance frontier of these factor mimicking portfolios intersects the mean variance frontier of r_{t+1} (see Huberman, Kandel & Stambaugh, (1987)). Given these factor mimicking portfolios, it was shown in Chapter 3 how regression based tests can be used to test for intersection in case r_{t+1} contains returns on futures contracts. These regression based intersection tests can also be extended to take restrictions on security positions into account (see Chapter 4).

However, in this chapter the interest is in analyzing specification errors in (6.5b) rather than testing whether the model is strictly valid. Therefore, in the empirical sections we focus on the effect of hedging pressure variables on futures risk premia and on estimating the specification error bounds δ. The limiting distribution of the estimated bound $\widehat{\delta}$ is given in Hansen, Heaton, & Luttmer (1995). There it is also shown that the fact that we have to estimate the parameters a, b, and c_s, $s = 1, ..., S$, does not affect the limiting distribution of $\widehat{\delta}$.

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23 In the empirical analysis of the specification error bounds in Section 6, r_{t+1} will include r^m_{t+1}, in which case we simply have $\text{Proj}\{r^m_{t+1} \mid r_{t+1}\} = r^m_{t+1}$.

6.4 Data

We use a dataset consisting of semimonthly observations of 20 futures contracts over the period January 1986 until December 1994. These futures contracts are divided into four categories, each containing five futures contracts: financial (S&P 500, Value-Line, T-Bond, T-Bill, Eurodollar), agricultural (wheat, corn, soybeans, live cattle, world sugar), mineral (gold, silver, platinum, crude oil, heating oil), and currency futures (Deutsche Mark, British Pound, Japanese Yen, Canadian Dollar, Swiss Franc). The composition of the dataset is comparable to the one studied by Bessembinder (1992). Details about the delivery months and the markets in which the futures contracts are traded can be found in Appendix 6.B. For this sample period we also have observations on positions of large traders in each of the futures contracts as reported by the Commodity Futures Trading Commission (CFTC). The S&P 500 Index will be used as a proxy for the market index. All data are obtained from the Futures Industry Institute Data Center. Continuous series of futures returns are created for each futures contract, for both the first and the second nearest-to-maturity contracts. These return series are created by using a roll-over strategy. For instance, for the nearest-to-maturity series a position is taken in the nearest-to-maturity contract until the delivery month, at which time the position changes to the following contract, which then becomes the nearest-to-maturity contract. To avoid the effect of the October 1987 crash, the returns in this month are excluded from the dataset. This results in a total of 40 series of 190 semi-monthly returns, two series for each futures contract. Summary statistics for the nearest-to-maturity series for all futures contracts are presented in Table 6.1. These summary statistics roughly confirm some well known stylized facts about futures returns. For instance, mean returns on agricultural and mineral futures are comparable in size with the mean returns on financial and currency futures, although for agricultural and mineral futures both positive and negative mean returns are observed. Standard deviations for agricultural and mineral futures returns are somewhat larger than for financial futures, but here it should be noted that the agricultural and mineral futures are based on individual commodities, whereas the financial futures are based either on equity portfolios or on interest rates.

The last two columns of Table 6.1 present the unconditional beta of each futures contract relative to the S&P 500 Index, along with the associated t-values. The t-values are based on heteroskedasticity consistent standard errors. Except for the financial futures, it is only for gold and silver futures contracts that we find betas that are significantly different from zero, and even for silver futures this is only marginally so. For all other agricultural, mineral and currency futures, the estimated betas are very small and never significantly different from zero. This indicates that

Table 6.1: Summary statistics for futures returns

The table contains summary statistics for semi-monthly returns on the nearest-to-maturity futures contracts in our sample. Returns are calculated for the period January 1986 until December 1994, excluding observations in the month October 1987. Mean returns and standard deviations are annualized ($\times 24$) and in percentages. The reported correlations are the average correlation of the futures contract with the five futures contracts in each group, where the futures contract itself is excluded in its own group. $\widehat{\beta}$ is the slope coefficient from an OLS regression of the futures returns on the S&P500 returns. Reported t-values for $\widehat{\beta}$ are based on heteroskedasticity consistent standard errors.

Financial futures								
	avg.	*stdv.*	*average correlations*				$\widehat{\beta}$	$t(\widehat{\beta})$
			Fin	Agr	Min	Cur		
S&P 500	11.10	13.70	0.454	0.009	-0.115	-0.015	1.028	(77.94)
Value Line	12.10	14.98	0.407	0.060	-0.115	-0.043	1.025	(21.10)
T-Bond	7.03	10.34	0.521	0.010	-0.216	0.049	0.377	(6.40)
T-Bill	0.21	1.23	0.403	-0.019	0.001	0.087	0.015	(1.92)
Eurodollar	0.72	1.20	0.529	-0.033	-0.096	0.106	0.031	(4.89)
Agricultural futures								
	avg.	*stdv.*	*average correlations*				$\widehat{\beta}$	$t(\widehat{\beta})$
			Fin	Agr	Min	Cur		
wheat	5.54	20.09	0.060	0.310	0.065	0.034	0.101	(0.83)
corn	-4.38	22.39	-0.012	0.342	0.010	-0.118	-0.020	(-0.16)
soybeans	0.31	20.34	-0.041	0.349	0.090	-0.050	-0.116	(-1.03)
live cattle	14.22	11.72	0.001	-0.020	0.038	0.058	0.061	(0.86)
world sugar	5.10	40.53	0.020	0.142	-0.050	0.008	0.103	(0.49)
Mineral futures								
	avg.	*stdv.*	*average correlations*				$\widehat{\beta}$	$t(\widehat{\beta})$
			Fin	Agr	Min	Cur		
gold	-4.07	13.51	-0.163	0.074	0.459	0.138	-0.258	(-2.84)
silver	-5.81	24.83	-0.145	0.155	0.341	0.012	-0.234	(-1.68)
platinum	-0.98	21.97	-0.011	0.086	0.323	0.115	-0.014	(-0.09)
crude oil	5.94	36.36	-0.125	-0.100	0.355	0.022	-0.436	(-1.53)
heating oil	16.45	36.44	-0.097	-0.063	0.275	0.055	-0.199	(-0.74)
Currency futures								
	avg.	*stdv.*	*average correlations*				$\widehat{\beta}$	$t(\widehat{\beta})$
			Fin	Agr	Min	Cur		
Deutsche Mark	4.70	12.12	0.063	-0.009	0.061	0.583	-0.014	(-0.18)
British Pound	4.52	11.95	0.021	-0.009	0.082	0.539	-0.043	(-0.68)
Japanese Yen	6.61	11.95	0.027	-0.028	0.065	0.479	-0.016	(-0.24)
Canadian Dollar	3.45	4.55	0.060	-0.001	0.050	0.023	0.036	(1.35)
Swiss Franc	3.80	12.92	0.012	-0.021	0.083	0.581	-0.076	(-1.03)

most commodity and currency futures in our sample do not have systematic risk, which confirms the results found by Dusak (1973), Carter, Rausser & Schmitz (1983) and Bessembinder (1992).

Finally, Table 6.1 also reports the average correlations of each futures contract with the futures contracts in the four groups, excluding the correlation of each contract with itself. For instance, the average correlation of the S&P 500 futures with the four other financial futures is 0.454, whereas the average correlation with the five agricultural futures is only 0.009. These average correlations show that the futures returns are highly correlated within each group but not across groups. With the exception of live cattle, sugar and Canadian Dollar futures the average correlations within each group are always above 0.25. On the other hand, the (absolute) average correlations across the four groups are much smaller. Because the returns on the assets underlying the futures contract and the nearest-to-maturity contract are usually very high and because reliable spot prices are hard to obtain, these correlations are used as a proxy for the correlation between the futures returns and the returns on the asset underlying the futures contract. Together with the fact that most futures contracts outside the financial groups have $\widehat{\beta}$'s close to zero, it follows from (6.6) that cross hedging pressure effects can be expected within each futures group but not between the groups. Therefore, in the next sections we will use a specification of the multifactor model in which the futures returns are related to the market return and to hedging pressure variables from within the futures own group, but not from the other groups.

In the models examined below, risk premia are related to hedging pressure variables. Positions of large traders in futures markets as reported by the Commodity Futures Trading Commission (CFTC) are used to construct proxies for the hedging pressures. Since large traders have to report to the CFTC whether they take a position in a futures market for hedging or for speculative reasons[24], these reports can be used to construct a variable that measures whether hedgers and speculators have a net long or short position in a futures market, which is essentially what a hedging pressure variable is supposed to measure. Therefore, for each futures contract s we create a variable $\widehat{q}^m_{s,t}$ that is based on reported positions of hedgers for each futures market s:

$$\widehat{q}^m_{s,t} = \frac{\textit{number of short hedge positions - number of long hedge positions}}{\textit{total number of hedge positions}}, \tag{6.13}$$

where the positions are measured by the number of contracts in market s. Notice that this variable takes a value between -1 and +1. Because it is believed that a net short (long) position of hedgers in a futures market creates a downward (upward) bias in the futures price, the variable $\widehat{q}^m_{s,t}$ as defined in (6.13) can be expected to have a positive relation with futures returns in market s.

Summary statistics for the hedging variable proxies are reported in Table 6.2. Notice that there can be quite some variation in the estimated hedging pressures. Not only is there substantial

24 Actually, the groups of traders are referred to as commercial and non-commercial traders, but this comes down to a distinction between hedgers and speculators (see also Bessembinder (1992)).

Table 6.2: Summary statistics for hedging pressure variables

The table contains summary statistics for hedging pressure variables based on the CFTC reports. The hedging pressure variable is defined as

$$\frac{(\textit{number of short hedge positions - number of long hedge positions})}{(\textit{total number of hedge positions})}.$$

Hedging pressures are calculated for the period January 1986 until December 1994, excluding observations in the month October 1987. Mean returns and standard deviations are in percentages. The reported correlations are the average correlation of the hedging pressure with the five hedging pressures in each group, where the futures own hedging pressure is excluded in its own group. $\widehat{\theta}$ is the slope coefficient from an OLS regression of the futures returns on their own hedging pressure. Reported t-values for $\widehat{\theta}$ are based on heteroskedasticity consistent standard errors.

Financial futures hedging pressures								
	avg.	*stdv.*	*average correlations*				$\widehat{\theta}$	$t(\widehat{\theta})$
			Fin	Agr	Min	Cur		
S&P 500	-6.7	6.1	0.013	0.066	-0.041	0.055	-0.019	(-0.53)
Value Line	0.3	52.9	-0.094	0.055	-0.094	0.198	-0.001	(-0.13)
T-Bond	-1.0	8.3	0.056	-0.153	0.041	-0.140	0.056	(2.87)
T-Bill	23.5	16.7	0.122	0.000	0.031	0.074	0.005	(3.97)
Eurodollar	-2.2	5.0	0.136	-0.135	-0.037	0.133	0.011	(2.60)
Agricultural futures hedging pressures								
	avg.	*stdv.*	*average correlations*				$\widehat{\theta}$	$t(\widehat{\theta})$
			Fin	Agr	Min	Cur		
wheat	-13.7	26.7	-0.101	0.077	0.040	-0.050	-0.025	(-2.24)
corn	83.5	5.7	0.058	-0.109	0.064	0.096	-0.193	(-3.39)
soybeans	-11.0	24.5	-0.119	0.013	0.214	-0.068	-0.019	(-1.60)
live cattle	25.4	15.0	0.045	-0.009	0.062	0.233	0.013	(1.07)
world sugar	23.6	18.1	-0.049	0.023	-0.026	0.214	0.137	(4.34)
Mineral futures hedging pressures								
	avg.	*stdv.*	*average correlations*				$\widehat{\theta}$	$t(\widehat{\theta})$
			Fin	Agr	Min	Cur		
gold	-0.2	20.5	-0.141	0.123	0.217	0.077	0.049	(5.47)
silver	39.4	11.7	-0.126	-0.063	0.001	-0.217	0.086	(2.81)
platinum	33.8	22.4	-0.050	0.199	0.170	0.113	0.054	(3.80)
crude oil	-2.1	6.8	0.188	0.017	0.147	-0.024	0.260	(2.48)
heating oil	6.7	9.9	0.030	0.077	0.145	0.057	0.226	(4.36)

variation in hedging pressure for a particular futures contract, as measured by the individual standard deviations, the cross-sectional differences between the hedging pressures appear to be quite large as well, as suggested by the differences in average hedging pressure. Also, whereas Keynes conjectured that it was "normal" for producers of agricultural commodities to be on the short side of the futures markets, whereas speculators would normally be on the long side, the statistics in Table 6.2 suggest that hedgers (and speculators) as a group can be either on the long or

Table 6.2: Summary statistics for hedging pressure variables (continued)

Currency futures hedging pressures								
	avg.	*stdv.*	*average correlations*				$\widehat{\theta}$	$t(\widehat{\theta})$
			Fin	Agr	Min	Cur		
Deutsche Mark	3.6	26.9	0.142	0.130	0.029	0.464	0.050	(10.10)
British Pound	1.2	42.7	0.041	0.057	-0.007	0.407	0.031	(9.45)
Japanese Yen	7.8	34.8	0.073	0.051	0.060	0.325	0.037	(8.62)
Canadian Dollar	15.8	45.8	-0.009	0.048	-0.118	0.030	0.009	(6.93)
Swiss Franc	2.7	39.9	0.072	0.139	0.042	0.456	0.035	(8.47)

on the short side of the market. This is also the case for the other groups of futures markets. Table 6.2 also reports the average correlations of each hedging pressure with the hedging pressures from each of the four groups. These correlations are somewhat different from the correlations between the futures returns in Table 6.1, in that the average correlations between the hedging pressures are usually low both within the futures own group and across the four groups. The exceptions are the hedging pressures for the currency futures and to some extent for the mineral futures, where the correlations within the futures own group are relatively high.

6.5 Analysis of hedging pressure and futures risk premia

As indicated by the model in Section 6.2, as well as previous models of Stoll (1979), Hirshleifer (1988a, 1989), and empirical work by Carter, Rausser & Schmitz (1983), Chang (1985), and Bessembinder (1992), hedging pressure variables are important determinants of expected futures returns. This also follows from the last two columns of Table 6.2, which show the slope coefficients and the associated t-values from a simple regression of futures returns on a constant and their own hedging pressure variable $\widehat{q}^{m}_{i,t}$. Except for the index futures and soybean and live cattle futures, there is always a significant relation between futures returns and hedging pressure. Also, except for wheat and corn futures, the coefficients that are significantly different from zero always have the expected positive sign. The interest here however, is not only in the relation between futures returns and the futures own hedging pressure, but also in the hedging pressure effects from other futures markets, i.e., in cross hedging pressure effects, as suggested by the model in (6.5b). To see the effects of hedging pressure from other futures markets on the futures risk premia, we will study each group of futures contracts, and analyze the effect of the hedging pressure variables within each group on futures returns. As indicated in Table 6.1, the correlations between futures contracts within the same group are usually rather high, whereas the opposite is true for the correlations between futures contracts from different groups. Also, futures contracts outside the financial group have $\widehat{\beta}$'s that are usually close to zero. These two results together suggest that $\theta_{s,i}$ in (6.6) will be close to zero if the nonmarketable risk s refers to another group than the futures contract i, implying that we may expect cross hedging pressure effects within each group but not between the four groups.

Denoting variables referring to futures contract i ($i = 1,..,5$) in group j ($j = 1,..,4$) as $x_i^{(j)}$, the regression model employed in this section is

$$r_{i,t+1}^{(j)} = \alpha_i^{(j)} + \beta_i^{(j)} r_{t+1}^{S\&P500} + \sum_{s=1}^{5} \theta_{i,s}^{(j)} \widehat{q}_{s,t}^{(j)} + \varepsilon_{i,t+1}^{(j)}. \tag{6.14}$$

This regression follows from a version of the ICAPM in which the nonmarketable risks in the 5 assets underlying the futures contracts in group j are the relevant state variables that agents want to hedge. Notice however, that if there are K futures contracts within each group, there are also K state variables, leading to a $K + 2$-factor model to explain K futures returns. This is not the case if we also include managed futures returns or if we differentiate futures contracts on some underlying asset with respect to their maturity for instance. In this section and the following we will use both the first and the second nearest-to-maturity futures contracts in the analysis.

Estimates of $\theta_{i,s}^{(j)}$ for the nearest-to-maturity contracts in each of the four groups of futures contracts are presented in Table 6.3. From this table it follows that, after accounting for market risk, the observed hedging pressure variables indeed have explanatory power for futures returns. Except for the S&P 500 Index futures and for soybean and live cattle futures, for each contract at least one of the hedging pressures within the own group results in an estimated coefficient $\widehat{\theta}_{i,s}^{(j)}$ that is significantly different from zero. Also, many contracts have significant coefficients for hedging pressures other than their own. For instance, all three metal futures show coefficients that are significantly different from zero for the silver and platinum hedging pressure variables. Similarly, except for Canadian Dollar futures, the hedging pressure for Deutsche Mark futures has a significant effect on all currency futures returns, consistent with cross hedging pressure effects.

The last two columns of Table 6.3 show test statistics for the hypotheses that (a subset of) the coefficients $\theta_{i,s}^{(j)}$ are equal to zero, using both the first and the second nearest-to-maturity futures contracts. If the hedging pressure variables are not important for the futures risk premia, all coefficients $\theta_{i,s}^{(j)}$ in (6.14) are equal to zero. The last-but-one column in the first panel of Table 6.3 shows Wald test-statistics for this hypothesis (W_{all}) along with the associated p-values. The reported test statistics leave little doubt about the relevance of hedging pressure variables in explaining futures returns. Except for soybean and live cattle futures, the hypothesis that all coefficients $\theta_{i,s}^{(j)}$ are zero can always be rejected. The last column shows Wald test statistics for the hypothesis that only the futures own hedging pressure variable is relevant, i.e., that $\theta_{i,s}^{(j)} = 0$, for

Table 6.3: Hedging pressure regressions

The table presents estimates of the coefficients $\theta_{s,i}^{(j)}$ in the regression

$$r_{i,t+1}^{(j)} = \alpha_i^{(j)} + \beta_i^{(j)} r_{t+1}^{S\&P500} + \sum_{s=1}^{5} \theta_{s,i}^{(j)} \widehat{q}_{s,t}^{(j)} + \varepsilon_{i,t+1}^{(j)},$$

where i refers to futures contract i in market j (financial, agricultural, mineral, currency). The variables $\widehat{q}_{s,t}^{(j)}$ are the 5 hedging pressure variables within the own market. $\theta_{s,i}^{(j)}$ therefore measures the sensitivity of the futures return to the hedging pressure variables in its own market group. All reported coefficients are $\times 100$. Values between brackets are t-values based on heteroskedasticity consistent standard errors. The regressions are reported for the nearest-to-maturity contracts. The last two columns present Wald test-statistics for the hypothesis that all reported coefficients are zero, $\theta_{s,i}^{(j)} = 0,\ \forall s, (= W_{all})$ and for the hypothesis that all reported coefficients except for the own hedging pressure variable are zero, $\theta_{s,j}^{(j)} = 0,\ s \neq i$ $(= W_{other})$. p-values are in brackets. The Wald test-statistics are based on regressions for both the nearest-to-maturity and second nearest-to-maturity contracts. All results are based on semimonthly observations over the period January 1986 until December 1994, excluding observations in the month October 1987.

Financial futures

	$\widehat{\theta}_{S\&P500}$	$\widehat{\theta}_{Value}$	$\widehat{\theta}_{TBond}$	$\widehat{\theta}_{TBill}$	$\widehat{\theta}_{Eur\$}$	W_{all}	W_{other}
S&P 500	-0.95	-0.00	-0.10	-0.14	-0.23	34.96	32.59
	(-1.25)	(-0.01)	(-0.18)	(-0.54)	(-0.32)	(0.000)	(0.000)
Value-Line	1.77	-0.22	1.12	-2.50	0.87	24.81	24.78
	(1.05)	(-0.98)	(0.58)	(-3.63)	(-1.07)	(0.006)	(0.002)
T-Bond	1.47	0.37	5.29	3.10	-3.34	57.31	31.18
	(0.45)	(1.01)	(2.75)	(3.65)	(-1.07)	(0.000)	(0.000)
T-Bill	-0.05	0.05	0.58	0.39	-0.02	87.92	7.11
	(-0.15)	(1.21)	(2.05)	(3.53)	(-0.06)	(0.000)	(0.525)
Eurodollar	0.05	0.06	0.76	0.50	-0.00	103.53	91.65
	(0.18)	(1.49)	(3.00)	(5.43)	(-0.01)	(0.000)	(0.000)

$i \neq s$ (W_{other}). This hypothesis can be rejected in 11 out of 20 times at the 5% significance level, indicating that there is also substantial evidence for the presence of cross hedging pressure effects.

Table 6.4 presents some further evidence on this issue by providing joint tests whether (a subset of) the coefficients $\theta_{i,s}^{(j)}$ are equal to zero for each group. The tests are based on both the first and second nearest-to-maturity contracts, so each test-statistic uses the returns on 10 futures contracts. The first panel shows again Wald test-statistics for the hypotheses that all coefficients $\theta_{i,s}^{(j)}$ are zero (W_{all}) and that all coefficients except for the futures own hedging pressure are zero respectively (W_{other}). The tests reported in Table 6.4 reject the null hypotheses at any conventional significance level, indicating that the futures own hedging pressure variables as well as cross hedging pressure variables within each group, have a significant effect on futures returns.

So far, we implicitly assumed that the measure for hedging pressure, $\widehat{q}_{s,t}^{m}$, coincides with the actual market hedging pressure. The hedging pressure variables $\widehat{q}_{s,t}^{m}$ used here however, are only

Table 6.3: Hedging pressure regressions (continued)

Agricultural futures							
	$\widehat{\theta}_{wheat}$	$\widehat{\theta}_{corn}$	$\widehat{\theta}_{soyb.}$	$\widehat{\theta}_{l.cttle}$	$\widehat{\theta}_{sugar}$	W_{all}	W_{other}
wheat	-3.53	1.32	1.36	-2.34	3.66	24.49	8.18
	(-3.08)	(0.26)	(1.13)	(-1.14)	(2.02)	(0.006)	(0.416)
corn	-1.63	-16.94	-1.16	-3.76	4.09	23.54	8.72
	(-1.09)	(-3.61)	(-0.80)	(-1.56)	(1.75)	(0.009)	(0.366)
soybeans	-0.91	-8.07	-1.58	-0.35	1.59	14.87	14.33
	(-0.80)	(-1.70)	(-1.08)	(-0.15)	(0.90)	(0.137)	(0.074)
live cattle	-1.24	1.81	0.36	1.14	-0.80	10.24	7.17
	(-1.36)	(0.67)	(0.43)	(0.89)	(-0.86)	(0.419)	(0.519)
world sugar	0.70	8.70	-3.83	-9.72	15.78	41.35	13.70
	(0.23)	(0.89)	(-1.36)	(-2.25)	(4.60)	(0.000)	(0.090)
Mineral futures							
	$\widehat{\theta}_{gold}$	$\widehat{\theta}_{silver}$	$\widehat{\theta}_{plat.}$	$\widehat{\theta}_{crude}$	$\widehat{\theta}_{heating}$	W_{all}	W_{other}
gold	2.88	3.03	2.03	2.74	2.56	58.05	19.36
	(2.81)	(1.92)	(2.16)	(0.87)	(1.28)	(0.000)	(0.013)
silver	2.10	8.07	5.22	-2.91	1.22	29.95	15.99
	(0.99)	(2.75)	(3.15)	(-0.51)	(0.35)	(0.001)	(0.043)
platinum	-1.63	6.20	6.51	-0.70	1.99	38.74	16.38
	(-1.09)	(2.44)	(4.37)	(-0.11)	(0.52)	(0.000)	(0.037)
crude oil	2.95	-0.07	-4.94	15.56	24.41	34.77	28.15
	(0.97)	(-0.02)	(-1.52)	(1.59)	(4.03)	(0.000)	(0.000)
heating oil	1.46	3.05	-4.29	12.99	19.93	35.51	6.98
	(0.42)	(0.74)	(-1.20)	(1.46)	(3.92)	(0.000)	(0.539)
Currency futures							
	$\widehat{\theta}_{DMark}$	$\widehat{\theta}_{Br.Pnd.}$	$\widehat{\theta}_{Jap.Y.}$	$\widehat{\theta}_{Can.\$}$	$\widehat{\theta}_{Sw.Fr.}$	W_{all}	W_{other}
Deutsche Mark	4.44	0.61	0.60	-0.66	-0.15	125.34	19.47
	(5.09)	(1.50)	(1.18)	(-1.83)	(-0.23)	(0.000)	(0.013)
British Pound	2.29	2.90	0.01	-0.06	-0.95	100.70	18.73
	(2.49)	(6.76)	(0.02)	(-0.17)	(-1.25)	(0.000)	(0.016)
Japanese Yen	1.99	0.07	3.13	-0.13	-0.33	90.53	11.26
	(1.89)	(0.16)	(6.31)	(-0.36)	(-0.48)	(0.000)	(0.187)
Canadian Dollar	-0.01	0.05	-0.21	0.91	0.14	62.03	10.05
	(-0.03)	(0.31)	(-1.19)	(6.63)	(0.63)	(0.000)	(0.261)
Swiss Franc	2.82	0.54	0.90	-0.65	1.17	130.58	24.41
	(2.91)	(1.21)	(1.71)	(-1.67)	(1.74)	(0.000)	(0.001)

proxies for the real market hedging pressure, i.e., we have an errors-in-variables problem. If it is assumed that the proxy is linear in the actual market hedging pressure, i.e.,

$$\widehat{q}^{m}_{s,t} = \varphi_{s,0} + \varphi_{s,1} q^{m}_{s,t} + u_{s,t},$$

Table 6.4: Joint Wald tests for zero hedging pressure effects from within each futures group

The table presents tests for the effect of hedging pressures within each futures own group, based on the regression

$$r_{i,t+1}^{(j)} = \alpha_i^{(j)} + \beta_i^{(j)} r_{t+1}^{S\&P500} + \sum_{s=1}^{5} \theta_{s,i}^{(j)} \widehat{q}_{s,t}^{(j)} + \varepsilon_{i,t+1}^{(j)},$$

where i refers to futures contract i in market (j) (financial, agricultural, mineral, currency). The variables $\widehat{q}_{s,t}^{(j)}$ are the 5 hedging pressure variables within the own market. W_{all} is a Wald test for the restriction that all $\theta_{s,i}^{(j)}$ within a group (j) are zero. W_{other} is a Wald test for the restriction that all coefficients $\theta_{s,i}^{(j)}$ are zero expect for each futures own hedging pressure variable. In the second part of the table estimates are based on Instrumental Variables estimates with lagged hedging pressure variables, lagged futures returns and the S&P 500 Index return as instruments. Wald test-statistics for the OLS-estimates are based on heteroskedasticity consistent covariance matrices, whereas the test-statistics for the IV-estimates are based on Newey-West estimates of the covariance matrix with a lag window of 1. Values in brackets are p-values. All results are based on semimonthly returns for the first and second nearest-to-maturity contracts in the sample, which is from January 1986 until December 1994, excluding observations in the month October 1987.

OLS-estimates, first and second nearest-to-maturity				
	W_{all}	(p-value)	W_{other}	(p-value)
Financial	263.27	(0.000)	225.98	(0.000)
Agricultural	131.38	(0.000)	71.62	(0.002)
Mineral	187.18	(0.000)	96.74	(0.000)
Currency	503.21	(0.000)	88.80	(0.000)
IV-estimates, first and second nearest-to-maturity				
	W_{all}	(p-value)	W_{other}	(p-value)
Financial	190.62	(0.000)	139.78	(0.000)
Agricultural	111.15	(0.000)	71.91	(0.001)
Mineral	95.62	(0.000)	68.85	(0.003)
Currency	107.22	(0.000)	44.33	(0.294)

it follows readily that the error term in (6.14) has a MA(1)-structure.

If the model in (6.5b) is valid however, it should still be the case that futures returns can be explained by the market hedging pressures $q_{s,t}^m$ after controlling for market risk. Therefore, we can still perform the tests presented in the first panel of Table 6.4, but given that we use the hedging pressure variable $\widehat{q}_{s,t}^m$ as a proxy for $q_{s,t}^m$, consistent estimates for the parameters in (6.14) can be obtained using an Instrumental Variables (IV) estimator, and the covariance matrix used in the Wald test-statistic should be corrected for the MA(1)-structure of the error terms, which can easily be accomplished by using a Newey-West estimator. The second panel of Table 6.4 presents Wald test statistics for the same two hypotheses as before, but now based on IV-estimates and consistent estimates of the covariance matrices. As instruments we use lagged hedging pressure variables $\widehat{q}_{s,t-1}^m$, as well as lagged futures returns $r_{i,t}$ and the return on the S&P 500 Index, $r_{t+1}^{S\&P500}$. The first two columns again show Wald test-statistics and the associated p-values for the hypothesis that all coefficients $\theta_{i,s}^{(j)}$ are zero. This hypothesis can always be rejected. The last two columns report these same statistics for the hypothesis that $\theta_{i,s}^{(j)} = 0$, for $i \neq 0$. For financial, agricultural and mineral futures the null hypothesis is still easily rejected. For currency futures however, the hypothesis that all coefficients $\theta_{i,s}^{(j)}$ except for the futures own hedging pressure are zero can not be rejected.

Summarizing, the evidence in this section shows that hedging pressure variables are important in explaining futures returns. Both the futures own hedging pressure and cross hedging pressures from within the futures own group appear to be relevant factors that should be included in a model as in (6.5b), after accounting for market risk. In the next section we will analyze the extent to which a multifactor model that uses hedging pressures from the futures own group as the relevant factors is misspecified, using the measure of misspecification of Hansen & Jagannathan (1997).

6.6 Specification error analysis for the multifactor model

The results in the previous section indicate that hedging pressure variables are important in explaining futures returns and that the futures own hedging pressure as well as cross hedging pressures from the futures own group should be included as factors besides market risk in the model in (6.5b). In this section we analyze how well the model in (6.5b) with hedging pressure variables from within the futures own group performs in pricing (portfolios of) futures contracts. To this end we employ the measure for model misspecification introduced by Hansen & Jagannathan (1997) which was discussed in Section 2.6 and in Section 6.3 of this chapter. In order to provide a benchmark, Table 6.5 reports estimates of the Hansen-Jagannathan specification error bounds, $\widehat{\delta}$, for the model in (6.5b) with no hedging pressure variables, in which case the model is identical to the simple (static) CAPM. Notice that in this case the proxy stochastic factor is of the form

$$y_{t+1} = a + br_{t+1}^{S\&P500}. \tag{6.15}$$

The parameters a and b are estimated in the minimization in (6.10). This implies that the estimated parameters a and b will in general be different for each group of futures contracts in Table 6.5, implying a different proxy stochastic discount factor for each group. Reporting specification error bounds for each group of contracts gives an indication for which group of futures the proxy in (6.15) is most misspecified. Table 6.5 also reports the bound for all futures markets together, in which case there is only one proxy stochastic discount factor for all markets.

Before discussing the results, one more remark should be made. Because futures contracts are zero-investment securities, it is not possible to determine the specification error bound for a set of securities consisting of futures only. At least one nonzero-investment security is needed.

Table 6.5: Specification error bounds for the model without hedging pressure

Specification error bounds are reported for a proxy stochastic discount factor of the form

$$y_{t+1} = a + br_{t+1}^{S\&P500}.$$

Specification error bounds $\widehat{\delta}$ are reported for the four groups of futures contracts (financial, agricultural, mineral, or currency) as well as for all futures contracts together. v is the mean of the discount factor. The standard errors are heteroskedasticity consistent. The specification error bounds are based on both the first and the second nearest-to-maturity contracts. Estimates are based on semi-monthly observations for the period January 1986 until December 1994, excluding observations in the month October 1987.

Nearest-to-maturity contracts			
	$\widehat{\delta}$	s.e.$(\widehat{\delta})$	$\widehat{\delta}/v$
Financial	0.132	0.074	0.131
Agricultural	0.237	0.084	0.234
Mineral	0.114	0.072	0.114
Currency	0.166	0.075	0.164
All	0.370	0.074	0.370
First and second nearest-to-maturity contracts			
	$\widehat{\delta}$	s.e.$(\widehat{\delta})$	$\widehat{\delta}/v$
Financial	0.269	0.070	0.269
Agricultural	0.296	0.074	0.295
Mineral	0.196	0.060	0.195
Currency	0.252	0.086	0.247
All	0.630	0.076	0.629

Therefore, we add to each set of futures returns the return on the S&P 500 Index. In effect this requires that the proxy stochastic discount factor does not only price the futures contracts correctly, but also the index. This seems to be a very weak assumption from an economic point of view, especially since the proxy itself is a function of the index.

The first part of Table 6.5, labelled *Nearest-to-maturity contracts,* presents the specification error bounds for returns on the nearest-to-maturity contracts only. For the four groups of futures contracts, the largest specification error bound is found for the agricultural futures (0.237) and the smallest for the mineral futures (0.117). Since the expectation of the discount factor is close to 1, the scaled specification error bound, $\widehat{\delta}/v$, is close to $\widehat{\delta}$. The estimated bounds may be compared with the bounds reported by Hansen & Jagannathan (1997) for six portfolios of stocks and bonds. Using a proxy that is also linear in the market portfolio and using monthly returns, Hansen & Jagannathan find a specification error bound of 0.286 with a standard error of 0.054. Since we use semi-monthly returns, the bounds in Table 6.5 can not immediately be compared with the bounds reported in Hansen & Jagannathan. However, given the interpretation of δ/v in terms of errors in Sharpe ratios, assuming the futures and market returns to be serially uncorrelated implies that our estimates of δ must be multiplied with $\sqrt{2}$ to get an approximation for the monthly bound. For the agricultural and mineral futures this means that the specification error bounds on a monthly basis are 0.335 and 0.165 with standard errors of 0.12 and 0.10 respectively. Therefore, the size of the bounds for the four groups of futures are comparable to the ones found by Hansen & Jagannathan (1997) for stock and bond portfolios, although the associated standard errors are twice as large.

Although the standard errors imply that the estimated specification error bounds may be somewhat imprecise, notice that the magnitude of the bounds is impressive in economic terms. For instance, even the smallest estimate of 0.117 for mineral futures implies that use of the CAPM may induce an error in the estimated Sharpe ratio of 0.117 on a semi-monthly basis for some portfolio of mineral futures and the S&P 500 Index. For a portfolio consisting of the S&P 500 Index and mineral futures with a semi-monthly standard deviation of 2.50%, which is comparable to the S&P 500, the CAPM could give an expected return that is as far off as 0.30% semimonthly (or 7.0% annually). For all futures together the estimated bound increases to 0.370 if simple futures returns are used. For comparison, the Sharpe ratio of the S&P 500 Index futures is equal to 0.165. Notice though, that this bound is only a maximum error that we can make for all available portfolios. For many portfolios that are of interest the error may be much smaller.

When the bounds are calculated for both the first and the second nearest-to-maturity contracts, there is a significant increase in the specification error bounds, as can be seen from the second part of Table 6.5. For instance, in case all futures are used, the specification error bound increases from 0.370 to 0.630, whereas the associated standard error hardly changes. For the four groups, the smallest bound is again obtained for the mineral futures and the largest bound for the agricultural futures.

To analyze whether the misspecification of the model in (6.5b) with hedging pressure variables from within the futures own group is smaller than the misspecification of the CAPM, Table 6.6 reports the specification error bounds for the model with hedging pressure variables. Notice that if only hedging pressure variables from within each futures group are used, the stochastic discount proxy is of the form

$$y_{t+1}^{(j)} = a^{(j)} + b^{(j)} r_{t+1}^{S\&P500} + \sum_{s=1}^{5} c_s^{(j)} \widehat{q}_{s,t}^{(j)}, \tag{6.16}$$

where we add superscripts (j) to indicate explicitly the fact that the proxy is different for each group j ($j = 1, ..., 4$). Table 6.6 also reports results for all futures markets together. In that case there are of course 20 hedging pressure variables in (6.16). The specification error bounds in Table 6.6 are reported for both the first and the second nearest-to-maturity contracts and should therefore be compared with the second panel in Table 6.5

Table 6.6: Specification error bounds for the model with cross hedging pressures

Specification error bounds are reported for a proxy stochastic discount factor of the form

$$y_{t+1} = a + br_{t+1}^{S\&P500} + \sum_{s=1}^{S} c_s \widehat{q}_{s,t}^{m},$$

where the hedging pressure variables are taken from the futures own group. Specification error bounds $\widehat{\delta}$ are reported for the four groups of futures contracts (financial, agricultural, mineral, or currency) as well as for all futures contracts together. v is the mean of the discount factor. The standard errors are based on Newey-West estimates with a lag window of 1. The specification error bounds are based on both the first and the second nearest-to-maturity contracts. Estimates are based on semi-monthly observations for the period January 1986 until December 1994, excluding observations in the month October 1987.

	$\widehat{\delta}$	$s.e.(\widehat{\delta})$	$\widehat{\delta}/v$
Financial	0.199	0.076	0.198
Agricultural	0.205	0.096	0.203
Mineral	0.098	0.080	0.096
Currency	0.157	0.149	0.152
All	0.421	0.109	0.420

The specification error bounds in Table 6.6 are certainly smaller than the ones in the second panel Table 6.5. For each group as well as for all futures contracts together, the estimated bounds, $\widehat{\delta}$, in Table 6.6 are 25%-50% smaller than the ones in the second panel of Table 6.5. The standard errors of the estimates are somewhat larger however. For each of the four groups the decrease in $\widehat{\delta}$ is in the order of magnitude of one standard error. For all futures together, the decrease in $\widehat{\delta}$ is more than two standard errors. Also, the decrease in the bounds is uniform over the four groups.

Some further insight on the bounds for both models can be obtained using the decomposition

$$\begin{aligned} \delta &= \| \widehat{y}_{t+1} - \widehat{m}_{t+1} \| = \sigma(\widehat{y}_{t+1} - \widehat{m}_{t+1}) \\ &= \{\sigma(\widehat{y}_{t+1})^2 + \sigma(\widehat{m}_{t+1})^2 - 2\rho_{ym}\sigma(\widehat{y}_{t+1})\sigma(\widehat{m}_{t+1})\}^{\frac{1}{2}}. \end{aligned}$$

Here $\widehat{m}_{t+1} = \text{Proj}\{m_{t+1} \mid r_{t+1}\}$, $\widehat{y}_{t+1} = \text{Proj}\{y_{t+1} \mid r_{t+1}\}$, and r_{t+1} is the vector of returns on the first and second nearest-to-maturity contracts in each futures group as well as the return on the S&P 500 Index. Recall that δ can be written as the standard deviation of $\widehat{y}_{t+1} - \widehat{m}_{t+1}$ because y_{t+1} is based on a factor model including a constant. Table 6.7 gives the standard deviations of $\widehat{y}_{t+1}$, $\widehat{m}_{t+1}$, as well as the correlation between those variables, ρ_{ym}, for both the model without hedging pressure variables in (6.15) and for the model with hedging pressure variables in (6.16).

The first panel of Table 6.7 gives the decomposition for the model without hedging pressure, i.e., the CAPM. For this model the volatility in $\widehat{y}_{t+1}$ is small relative to the volatility in $\widehat{m}_{t+1}$, i.e., the minimum volatility that is needed for any valid stochastic discount factor. Therefore, for the model without hedging pressure, the size of $\widehat{\delta}$ is mainly determined by $\sigma(\widehat{m}_{t+1})$. The second

Table 6.7: Decomposition of the specification error bounds

The first panel of the table shows the decomposition of the specification error bounds for the model without hedging pressure, the second panel for the model with cross hedging pressures from within the futures own group (financial, agricultural, mineral, or currency). The components of the specification error bound $\widehat{\delta}$ are the standard deviations $\sigma(\widehat{m}_{t+1})$, $\sigma(\widehat{y}_{t+1})$,and the correlation ρ_{ym}, where $\widehat{m}_{t+1} = \text{Proj}\{m_{t+1} \mid r_{t+1}\}$, $\widehat{y}_{t+1} = \text{Proj}\{y_{t+1} \mid r_{t+1}\}$, and r_{t+1} is the vector of returns on the first and second nearest-to-maturity contracts in each futures group as well as the return on the S&P 500 Index. Estimates are based on semi-monthly observations for the period January 1986 until December 1994, excluding observations in the month October 1987.

$y_{t+1} = a + br_{t+1}^{S\&P500}$			
	$\sigma(\widehat{y}_{t+1})$	$\sigma(\widehat{m}_{t+1})$	ρ_{ym}
Financial	0.174	0.321	0.542
Agricultural	0.077	0.307	0.252
Mineral	0.108	0.225	0.483
Currency	0.251	0.356	0.705
All	0.185	0.658	0.281
$y_{t+1} = a + br_{t+1}^{S\&P500} + \sum_{s=1}^{5} c_s q_{s,t}^m$			
	$\sigma(\widehat{y}_{t+1})$	$\sigma(\widehat{m}_{t+1})$	ρ_{ym}
Financial	0.252	0.321	0.785
Agricultural	0.228	0.307	0.742
Mineral	0.202	0.225	0.900
Currency	0.319	0.356	0.896
All	0.505	0.658	0.768

panel of Table 6.7 gives the same decomposition for the multifactor model with hedging pressure variables from within the futures own group. For this model the volatility of $\widehat{y}_{t+1}$ is much higher than for the model without hedging pressure. $\sigma(\widehat{y}_{t+1})$ is now relatively close to $\sigma(\widehat{m}_{t+1})$, and the adjustment that is necessary to give $\widehat{y}_{t+1}$ the necessary variability, δ, is accordingly smaller.

Taken together, the evidence in this section indicates that models including hedging pressure variables do perform better than the model without hedging pressure, both in statistical and economic terms. The estimated bounds for the model without hedging pressure are comparable to the ones reported by Hansen & Jagannathan (1997) for stock and bond portfolios. Although the estimated bounds appear to be rather large, implying that some portfolios of the S&P 500 Index and the futures contracts may be severely mispriced, this mispricing can be reduced significantly if cross hedging pressure variables are taken into account.

6.7 Summary and conclusions

In this chapter we presented a simple multifactor model in which futures risk premia are determined by the covariance of the futures return with the market return, as well as by hedging pressure variables. The model does not only identify the futures own hedging pressure as a determinant of the futures risk premium, but also hedging pressure from other futures markets, referred to as cross hedging pressures. For a set of 20 futures markets, that are divided into four groups (financial, agricultural, mineral, and currency futures), we analyze the model using both standard test procedures as well as the specification error bounds introduced by Hansen & Jagannathan (1997). We show that the futures own hedging pressure as well as cross hedging pressures from within the same group have a significant effect on the futures returns after controlling for market risk. We also show that the inclusion of hedging pressure variables induces a substantial reduction in the estimated specification error bounds. However, the estimated specification error bounds are rather large both for the model without hedging pressure and for the model with hedging pressure.

Appendix 6.A Derivation of the equilibrium model

In this appendix we will show that if the market portfolio w^m is multifactor efficient (ME), then (6.5) follows from (6.3). Notice that the market portfolio is of the form $w^m = (w'_A\ 0')'$, i.e., futures contracts do not enter the market portfolio since they are in zero net supply. Let the optimal portfolio of agent j as derived from (6.3) be denoted by w^j, and the agents wealth invested in financial markets (which may deviate from his total wealth because of the agents' nonmarketable positions) by Y_t^j. The market portfolio is then defined by

$$w^m = \frac{\sum_{j=1}^N Y_t^j w^j}{\sum_{j=1}^N Y_t^j},$$

and the aggregate market risk aversion γ^m and aggregate nonmarketable positions $q_{s,t}^m$ by

$$\gamma^m = \left(\frac{\sum_{j=1}^N Y_t^j (\gamma^j)^{-1}}{\sum_{j=1}^N Y_t^j}\right)^{-1} \text{ and } q_{s,t}^m = \frac{\sum_{j=1}^N Y_t^j q_{s,t}^j}{\sum_{j=1}^N Y_t^j}.$$

Now suppose that the market portfolio w^m is ME. Augment the vector r_{t+1} with the return on the market portfolio, i.e., the augmented return vector $\widetilde{r}_{t+1} = (r_{t+1}^m\ r'_{t+1})'$, and define $\widetilde{w}$ in a similar way. For simplicity we assume that all variances and covariances are time-invariant. If the market portfolio is ME, then there must exist an optimal portfolio $\widetilde{w}^*$ which is of the form $(\omega^m\ 0'_{K+L})'$, with $\omega^m = 1$, and the three parts of $E_t[\widetilde{r}_{t+1}]$ that correspond with the market, all assets, and all futures respectively, can be written as

$$\begin{aligned}
E_t[r_{t+1}^m] - \eta &= \gamma^m\{Var[r_{t+1}^m]\omega^m + Cov[r_{t+1}^m, r_{S,t+1}]q_t^m\}, && \text{(A.1a)}\\
E_t[r_{A,t+1}] - \eta\iota &= \gamma^m\{Cov[r_{A,t+1}, r_{t+1}^m]\omega^m + Cov[r_{A,t+1}, r_{S,t+1}]q_t^m\}, && \text{(A.1b)}\\
E_t[r_{F,t+1}] &= \gamma^m\{Cov[r_{F,t+1}, r_{t+1}^m]\omega^m + Cov[r_{F,t+1}, r_{S,t+1}]q_t^m\}. && \text{(A.1c)}
\end{aligned}$$

Substituting ω^m from (A.1a) into (A.1b) and (A.1c), gives the required result:

$$\begin{aligned}
E_t[r_{A,t+1}] - \eta\iota &= \beta_A E_t[r_{t+1}^m - \eta] + \sum_{s=1}^S \theta_{A,s} q_{s,t}^m,\\
E_t[r_{F,t+1}] &= \beta_F E_t[r_{t+1}^m - \eta] + \sum_{s=1}^S \theta_{F,s} q_{s,t}^m.
\end{aligned}$$

with

$$\begin{aligned}
\beta_i &= Cov[r_{i,t+1}, r_{t+1}^m]Var[r_{t+1}^m]^{-1},\\
\theta_{i,s} &= \gamma^m\{Cov[r_{i,t+1}, r_{s,t+1}] - \beta_i Cov[r_{t+1}^m, r_{s,t+1}]\}.
\end{aligned}$$

Appendix 6.B Futures data

In this appendix we provide some additional details about the futures contracts used in this chapter. For all futures contracts the exchange at which they are traded is given, as well as a list of the delivery months.

Contract	*Exchange*	*Delivery months*
Financial		
S&P 500	Chicago Mercantile Exchange	3, 6, 9, 12
Value Line	Kansas City Board of Trade	3, 6, 9, 12
T-Bond	Chicago Board of Trade	3, 6, 9, 12
T-Bill	Chicago Mercantile Exchange	3, 6, 9, 12
Eurodollar	Chicago Mercantile Exchange	3, 6, 9, 12
Agricultural		
wheat	Chicago Board of Trade	3, 5, 7, 9, 12
corn	Chicago Board of Trade	3, 5, 7, 9, 12
soybeans	Chicago Board of Trade	1, 3, 5, 7, 8, 9, 11
live cattle	Chicago Mercantile Exchange	2, 4, 6, 8, 10, 12
world sugar	Coffe, Sugar and Cacoa Exchange	3, 5, 7, 10
Mineral		
gold	Commodity Exchange, Inc.	2, 4, 6, 8, 10, 12
silver	Commodity Exchange, Inc.	1, 3, 5, 7, 9, 12
platinum	New York Mercantile Exchange	1, 4, 7, 10
crude oil	New York Mercantile Exchange	*All*
heating oil	New York Mercantile Exchange	*All*
Currency		
Deutsche Mark	Chicago Mercantile Exchange	3, 6, 9, 12
British Pound	Chicago Mercantile Exchange	3, 6, 9, 12
Japanese Yen	Chicago Mercantile Exchange	3, 6, 9, 12
Canadian Dollar	Chicago Mercantile Exchange	3, 6, 9, 12
Swiss Franc	Chicago Mercantile Exchange	3, 6, 9, 12

Chapter 7

Pricing Term Structure Risk in Futures Markets

7.1 Introduction

In the literature on both financial and commodity futures markets[25] a large body of empirical evidence exists that futures prices differ from expected future spot prices because of risk premia (e.g. Fama (1984a), Fama & French (1987), Bessembinder (1992)). Investors expect to earn these spot-futures premia by taking positions in the futures market and holding these until the maturity date of the contracts. On the other hand, expected returns on futures positions that are not held until the delivery date, do not only depend on these spot-futures premia but also on the risk premia in futures spreads. Risk premia in futures spreads cause differences in the expected one-period returns on futures contracts with different maturities. The aim of this paper is to analyze risk premia in futures contracts with different maturities. As in Chapter 5, we define the annualized spread between the spot and futures price as the *yield*. By a no-arbitrage argument the yield is the difference between the interest rate and the net cash flow that accrues to the marginal owner of the asset. These net cash flows consist e.g. of dividends, foreign interest rates, and convenience yields, net of storage costs. Long maturity yields can be decomposed in an expected future short maturity yield plus a risk premium, in the same way as long interest rates can be decomposed in expected future short interest rates and a liquidity premium. These risk premia in the term structure of yields are equal to the differences in one-period risk premia on futures contracts with different maturities. We will maintain this dual interpretation of the risk premia throughout the chapter.

Our analysis focuses on the information that is present in the current term structure of futures prices with respect to expected future yields and risk premia in the yields. This is similar to the analysis of forward currency, interest, and commodity markets in Fama (1984a,b,c) and Fama & French (1987). However, these papers focus on the predictive power of futures prices for future spot prices and on spot-futures risk premia. The approach in this chapter is different, because we study the differences in one-period risk premia between futures contracts with different maturities.

[25] We will ignore the difference between futures and forward contracts. For a detailed discussion of the difference between these two types of contracts, see, e.g., Cox, Ingersoll, & Ross (1981).

A related part of the literature focuses on the relation between yields and spot price changes (see e.g. Fama & French (1988), Bessembinder et al. (1995)), but it does not consider the yields and yield changes themselves. The aim here is to fill this gap. Our analysis of yields is closely related to the analysis of interest rates in Backus, Foresi, Mozumdar and Wu (BFMW) (1997) and to the analysis of commodity futures pricing in Schwartz (1997).

Pricing forward and futures contracts for maturities that are not (yet) traded, or pricing other derivative securities on the assets underlying the futures contracts, requires knowledge about the covariance of the pricing kernel and the yields. These covariances can be derived from a simple affine one-factor model for the yields, that encompasses both a Vasicek and a Cox-Ingersoll-Ross (CIR) like model as special cases. It is only in the special case that the Vasicek-like model is valid, that the covariances of the kernel with the yields are constant. This implies that risk premia are constant and that the term structure of futures prices contains no information about risk premia in the yields. With a CIR-like specification the covariance of the kernel and the yields are dependent on the current level of the short yield, implying that the risk premia depend on the current slope of the term structure of futures prices.

The empirical analysis is conducted for five futures contracts: gold, heating oil, live cattle, soybeans, and Deutsche Mark futures contracts. We use observations both at a low frequency, which is equal to the frequency of the delivery dates of the specific contract, and at a daily frequency. The results show that for heating oil and Deutsche Mark futures the data are consistent with a Vasicek-like model for the term structure of yields. For heating oil we can not reject the hypothesis that the risk premia in the term structure of yields are constant. Also, for these contracts the estimated risk premia are always negative. This implies that one-period expected returns on heating oil futures are lower for the longer maturity contracts and that the term structure of yields is upward sloping. For Deutsche Mark futures we can not reject the hypothesis that the premia are zero. Finally, gold and soybean futures show evidence that risk premia depend on the current slope of the futures term structure, while the evidence for live cattle futures is mixed. The evidence for soybean futures shows that our one-factor model has the same kind of difficulty in explaining both the regression evidence and the mean slope of the term structure, as one-factor models for interest rates (BFMW (1997)). The empirical results in this chapter have clear implications for hedging and portfolio decisions, as well as for pricing other derivative securities.

The outline of this chapter is as follows. In Section 7.2 we will show how to derive information about risk premia from the term structure of futures prices. Section 7.3 shows the implications of a simple one-factor model for the term structure of futures prices. In Section 7.4 we will present the empirical analysis. Finally, Section 7.5 contains the concluding remarks.

7.2 The information in futures prices with different maturities

When buying an asset on the spot market at time t and simultaneously taking a short position in the futures market for delivery at time $t + n$, an investor can lock in a certain return if both the spot and the futures position are held until maturity[26]. We will refer to this certain return as the (continuously compounded) *yield*, $y_t^{(n)}$:

$$y_t^{(n)} \equiv \frac{f_t^{(n)} - s_t}{n}, \tag{7.1}$$

where $f_t^{(n)}$ is the log futures price for delivery at $t + n$, and s_t is the log spot price. This yield is also known as the *annualized spread* or the *slope* of the futures term structure. By a no arbitrage argument, the yield is equal to the n-period interest rate minus the net cash flow (as a percentage of the spot price) that accrues to the marginal owner of the asset. This net cash flow consists for instance of dividends, foreign interest rates, or convenience yields, net of any storage costs that have to be paid for holding the asset. Similarly, the *forward yield* (or *annualized forward spread*), $h_t^{(k,n)}$, is the yield that an investor can earn from time $t + k$ to $t + n$, which he can lock in at time t by taking simultaneously a long position in a futures contract that matures at $t + k$, and a short position in a futures contract that matures at $t + n$:

$$h_t^{(k,n)} \equiv \frac{f_t^{(n)} - f_t^{(k)}}{n-k} = \frac{ny_t^{(n)} - ky_t^{(k)}}{n-k}. \tag{7.2}$$

It is obvious that the term structure of (forward) yields can be derived from spot and futures prices in the same way that the term structure of (forward) interest rates can be derived from bond and bond futures prices.

Focussing on one-period changes in spot and futures prices, the forward yield $h_t^{(1,n)}$ can be decomposed in the expected future yield $E_t[y_{t+1}^{(n-1)}]$ and a risk premium $\theta_t^{(n)}$:

$$h_t^{(1,n)} \equiv \frac{ny_t^{(n)} - y_t^{(1)}}{n-1} = E_t[y_{t+1}^{(n-1)}] + \frac{\theta_t^{(n)}}{n-1}. \tag{7.3}$$

In terms of spot and futures prices, equation (7.3) can be rewritten as:

$$\theta_t^{(n)} = E_t[s_{t+1} - f_t^{(1)}] - E_t[f_{t+1}^{(n-1)} - f_t^{(n)}]. \tag{7.4}$$

This equation shows that the risk premium $\theta_t^{(n)}$ equals the expected one period return on a spreading strategy that involves a long position in a futures contract with one period to maturity and a short position in a futures contract with n periods to maturity. Alternatively, $-\theta_t^{(n)}$ is the expected one

[26] We will ignore transaction costs that are associated with possible delivery.

period return on a futures contract with n periods to maturity, in excess of the return on a one period futures contract. Thus, the different premia $\theta_t^{(n)}$ that are present in the term structure of yields, also show up as the differences in the one period expected returns on futures contracts with different maturities.

Although the risk premia $\theta_t^{(n)}$ ultimately arise from uncertainty in the *yields*, i.e., in dividends, convenience yields, etc., Equation (7.4), which is in terms of spot and futures *prices*, provides a convenient way of communicating empirical results with respect to the term structure. Equation (7.4) shows that the forward spread between the n-period futures price and the 1-period futures price, $f_t^{(n)} - f_t^{(1)}$, contains information about next period's $(n-1)$-period spread, $f_{t+1}^{(n-1)} - s_{t+1}$, and about the risk premium $\theta_t^{(n)}$. It is well known from a series of papers by Fama (1984a,b,c, 1986) that the extent to which variation in both the future spread $f_{t+1}^{(n-1)} - s_{t+1}$, and variation in the risk premium $\theta_t^{(n)}$ show up in the variance of the forward spread $f_t^{(n)} - f_t^{(1)}$ can be analyzed by the complementary regressions:

$$f_{t+1}^{(n-1)} - s_{t+1} = \alpha_1 + \beta_1(f_t^{(n)} - f_t^{(1)}) + \eta_{1,t+1}, \tag{7.5a}$$

$$(s_{t+1} - f_t^{(1)}) - (f_{t+1}^{(n-1)} - f_t^{(n)}) = \alpha_2 + \beta_2(f_t^{(n)} - f_t^{(1)}) + \eta_{2,t+1}. \tag{7.5b}$$

The error term $\eta_{1,t+1}(= -\eta_{2,t+1})$ is the prediction error of next period's spread, i.e. $\eta_{1,t+1} = (f_{t+1}^{(n-1)} - s_{t+1}) - E_t[f_{t+1}^{(n-1)} - s_{t+1}]$. The first regression in (7.5) answers the question whether forward spreads have power to predict future spreads. If this is the case, this will result in an estimate of β_1 which is different from zero. The predictive power of forward spreads for future spreads is diminished if there is variation in the risk premia $\theta_t^{(n)}$ that shows up in the forward spread. This will result in the estimate of β_2 in the second regression in (7.5) being different from zero (and the estimate of β_1 being different from one).

The analysis presented in this section isolates the information that is present in the term structure of futures prices with respect to expected future yields and risk premia in yields. Earlier studies concentrated on the information in futures prices with respect to future spot prices and risk premia in spot market returns (e.g. Fama (1984a,b,c, 1986), Fama & French (1987)) or on the interaction of yields and spot prices (e.g. Fama & French (1988), Bessembinder et al. (1995, 1996)), rather than on the yields themselves, as we do here. In the next section we will show that in a simple one-factor model for the term structure of yields, the covariances of the yields with the pricing kernel can be derived from the term structure of futures prices. Knowledge of these covariances can be used to price many other derivative securities on the asset underlying the futures contracts. The one-factor model will also provide testable implications for the coefficients α and β in (7.5).

7.3 A one-factor model for the term structure of yields

7.3.1 An affine one-factor model

In equilibrium, in frictionless markets, the risk premium $\theta_t^{(n)}$ is determined by the covariance of $y_{t+1}^{(n-1)}$ and the stochastic discount factor, or pricing kernel, m_{t+1}:

$$\theta_t^{(n)} = Cov_t[m_{t+1}, (n-1)y_{t+1}^{(n-1)}]. \tag{7.6}$$

The stochastic discount factor is known to be proportional to the marginal (derived) utility of rational agents, given their optimal portfolio and consumption choice (see e.g. Ingersoll (1987)). It is straightforward to show that equation (7.6) follows from the first order conditions of the portfolio and consumption problem. Using a suitable specification of the process for the short yield, $y_t^{(1)}$, it is possible to characterize the covariances of the pricing kernel with the yields and to make testable statements about the term structure of futures prices. In this section we will show the implications of an affine one-factor model for the term structure of yields. This discussion closely follows the one-factor models for the term structure of interest rates as outlined for instance in Campbell, Lo & McKinlay (1997).

Assume that the short yield, $y_t^{(1)}$, follows a first order autoregressive process, with possibly heteroskedastic innovations, of the following form:

$$y_{t+1}^{(1)} = \mu + \rho(y_t^{(1)} - \mu) + ((1-\omega) + \omega y_t^{(1)})\varepsilon_{t+1}. \tag{7.7}$$

Here $0 \leq \omega \leq 1$ and ε_{t+1} is an *i.i.d.* random variable with $E_t[\varepsilon_{t+1}] = 0$, and $Var_t[\varepsilon_{t+1}] = \sigma(\varepsilon)^2$. We will also assume that the covariance of ε_{t+1} with the stochastic discount factor is constant, i.e. $Cov_t[\varepsilon_{t+1}, m_{t+1}] = \sigma_{\varepsilon m}$. Notice that because of the possible heteroskedasticity in $y_{t+1}^{(1)}$ this does not imply that the covariance of the pricing kernel with the yields is constant. If $\omega \neq 0$, then both the variance of $y_{t+1}^{(1)}$ and the covariance of $y_{t+1}^{(1)}$ with m_{t+1} depend on $y_t^{(1)}$, implying time-varying risk premia similar to the risk premia in the CIR-model for interest rates. Although the process for $y_t^{(1)}$ is exogenously given here, (7.7) may very well be the reduced form of a model in which $y_t^{(1)}$ is the (endogenous) result of the optimal decisions about consumption, production, and storage made by rational agents. Since here the aim is to derive the information about risk premia and future yields that is present in the term structure of futures prices, we take the model for the short yield as given.

The process in (7.7) encompasses two special cases if ω is either 0 or 1. If $\omega = 0$, the process in (7.7) is similar to the process for the short term interest rate specified by Vasicek (1977). In this case, (7.7) is also similar to the model for the convenience yield in Brennan (1991), referred to as the *autonomous convenience yield* model. This model is also closely related to the model for commodity futures prices analyzed by Schwartz (1997), with a mean reverting convenience yield and constant interest rates. If $\omega = 1$ then we obtain a specification that is similar to the interest rate process specified by Cox, Ingersoll, & Ross (CIR) (1985). If $0 < \omega < 1$, a mixture of the two processes is obtained.

Substituting the AR(1) model in (7.7) into (7.3) and (7.6), we can solve all yields and risk premia as functions of the short term yield. The yield for any maturity can be written as:

$$y_t^{(n)} = \frac{1}{n} A^{(n)} + \frac{1}{n} B^{(n)} y_t^{(1)},$$

with:

$$\begin{aligned} A^{(n)} &= A^{(n-1)} + \left((1-\rho)\mu + (1-\omega)\sigma_{\varepsilon m}\right) B^{(n-1)} \\ B^{(n)} &= \frac{1-(\rho+\omega\sigma_{\varepsilon m})^n}{1-(\rho+\omega\sigma_{\varepsilon m})}, \end{aligned} \tag{7.8}$$

and $A^{(0)} = B^{(0)} = 0$. Similarly, the covariances of the yields with the pricing kernel, or the risk premia, are given by:

$$\theta_t^{(n)} = B^{(n-1)}((1-\omega) + \omega y_t^{(1)})\sigma_{\varepsilon m}. \tag{7.9}$$

Thus, all yields and risk premia are affine functions of a single factor, the short yield $y_t^{(1)}$[27]. In the special case that $\omega = 0$, i.e., when the process for the short yield is homoskedastic, the risk premia do not depend on $y_t^{(1)}$, but are constant for each value of n. For all other values of ω the risk premia will be time-varying, where all variation is captured by $y_t^{(1)}$.

In the model presented here, knowing only the spot price and one futures price (i.e., one spread), in principle allows us to determine the complete term structure of futures prices at a given date, since both the expected future yields and the covariances of the kernel with the yields depend on the short yield only. As stated before, this term structure can then be used in pricing other derivative securities on the asset underlying the futures contract. For instance, in valuing European options on an asset, under a risk neutral measure the expected spot price of the asset at the maturity date of the option will typically be replaced by the futures price of the asset, for the same maturity as the option. If a futures contract on the asset with the same maturity as the option is not traded, the one-factor model can provide the necessary input to determine the option price. This is true for many other derivative securities as well[28].

[27] Of course it is also possible to derive the model starting from a continuous version of the process for the short yield, as in Vasicek (1977) and Cox, Ingersoll, & Ross (1981). This would only affect the constant terms in Equations (8) and (9), but not the slope coefficients.

[28] Similarly, Carr & Jarrow (1995) present a framework to price derivative securities based on binomial trees, that starts from the term structure of futures prices.

7.3.2 Empirical implications of the one-factor model

In the one-factor model for the term structure of yields presented above, all variation in (expected) yields and risk premia is due to variation in one factor, which may be the short yield. Since the regressions in (7.5) are specified in terms of spreads (yields) only, the regression parameters are fully determined by the model parameters given above. In particular, if the one-factor model is valid, the slope coefficients β_1 and β_2 from (7.5) can be written as:

$$\beta_1 = \frac{\rho}{\rho + \omega\sigma_{\varepsilon m}}, \quad \beta_2 = \frac{\omega\sigma_{\varepsilon m}}{\rho + \omega\sigma_{\varepsilon m}}, \tag{7.10}$$

while the constant α_1 ($\alpha_2 = -\alpha_1$) is given by:

$$\alpha_1 = B^{(n-1)}(1-\omega)\sigma_{\varepsilon m} + \beta_2 A^{(n)}. \tag{7.11}$$

For one thing, these solutions show that in the one-factor model the slope coefficients in (7.5) do not depend on the maturity n, but are the same for all spreads along the term structure. Differences in maturity only show up in the intercepts. The absolute values of the intercepts are increasing functions of the maturity. A similar analysis relating affine models of interest rates to regressions similar to (7.5) is given by BFMW (1997).

If the process of the short yield is homoskedastic, i.e., $\omega = 0$, and if $\rho \neq 0$, the slope coefficient β_1 will be equal to one, and β_2 will be equal to zero. Thus, if the short yield follows a Vasicek-like process with $\rho \neq 0$, i.e., the short yield is not expected to revert to its long term average immediately, all variation in the forward spreads is due to variation in expected future spreads, which could be expected since the homoskedastic model implies constant risk premia. If $\omega = 0$ and $\rho \neq 0$, the intercept α_2 gives a direct estimate of the risk premium $\theta^{(n)}$.

The opposite extreme case where β_1 is zero and β_2 is equal to one, is obtained if $\rho = 0$ and $\omega\sigma_{\varepsilon m} \neq 0$. In this situation expected future short yields do not depend on the current short yield, but are constant. Therefore, current spreads do not contain any information about expected future spreads, implying that β_1 will indeed be equal to zero. The condition that $\omega\sigma_{\varepsilon m} \neq 0$ means that the short yield is heteroskedastic and therefore that variation in the short yield does cause variation in the risk premia. This variation naturally shows up in the coefficient β_2.

Summarizing, the fact that $\rho \neq 0$ implies that expected future short yields depend on the current short yield. This shows up in the slope coefficient β_1 in (7.5) being different from zero, i.e. in the forward spread having predictive power for future spreads. The fact that $\omega\sigma_{\varepsilon m} \neq 0$ implies that (co)variances of the yields $y_{t+1}^{(n)}$ depend on the current level of the short yield, $y_t^{(1)}$. This is also true for the covariance of the kernel m_{t+1} with the yields, resulting in time-varying risk premia that are affine functions of the current short yield. This shows up in the slope coefficient β_2 in (7.5) being different from zero, i.e. in the forward spread having predictive power for excess returns.

7.4 Empirical results

7.4.1 Description of the data

In the previous two sections it was shown how information about risk premia and future spreads could be obtained from the current futures term structure and what the implications of a simple one-factor model for the futures term structure are. In this section we will analyze the futures term structure for five contracts: gold, Deutsche Mark, heating oil, live cattle, and soybean futures. The starting point of the analysis will be Equation (7.5).

Since Equation (7.5) requires that in every period we observe at least futures contracts with one period to maturity and one other maturity, the observation frequency of the futures contracts can not exceed the delivery frequency. As not every month is a delivery month for all futures contracts considered, the observation frequency is different for the respective futures contracts. For instance, gold futures contracts are traded for delivery in February, April, June, August, October, and December. Therefore, the observation frequency for gold futures is once every two months and *one period* refers to two months for gold futures. Table 7.1 contains summary statistics for the yields of the five futures for several maturities. Column 2 of Table 7.1 contains information about the length of *one period.* In Section 7.4.2 we will present empirical results for the futures term structure based on these low frequency data. In Section 7.4.3 a similar analysis will be given based on daily data.

We use data from the Futures Industry Institute for the period starting from March 1970 or from the start of trading in the contract, until December 1993 . Because for many commodities futures prices are more reliable than spot prices, we use the futures price at the delivery date as the spot price, rather than the spot price itself.

From the summary statistics in Table 7.1, we see that the mean short term yield $y_t^{(1)}$ is negative for heating oil and live cattle, whereas it is positive for gold, soybeans and Deutsche Marks. Given positive interest rates and storage costs, this means that for heating oil and live cattle the average one-period convenience yield must be larger than the sum of the interest rate and storage costs. In case of Deutsche Mark futures, the positive mean yield implies that US-interest rates must on average have been higher than German interest rates by approximately 3% per year. Comparing the mean yields for different maturities shows that the average yield curve is upward sloping for gold, heating oil and live cattle, and downward sloping for soybeans. Assuming that the average

Table 7.1: Summary statistics for yields

The table contains summary statistics for yields on futures contracts which are defined as the annualized spread between the spot and futures price. Observations are from March 1970, or from the beginning of trading in the contract, until December 1993. Average, standard deviation, maximum, and minimum are in percentage per period, where the length of one period is indicated in the first column.

Contract		One period	Average	Std.dev.	Minimum	Maximum	AR(1)
gold	$y_t^{(1)}$	2 months	0.71%	0.34%	0.01%	1.72%	0.75
($N = 114$)	$y_t^{(2)}$		1.03%	0.47%	0.21%	2.46%	0.89
	$y_t^{(3)}$		1.15%	0.52%	0.28%	2.78%	0.91
	$y_t^{(4)}$		1.21%	0.53%	0.31%	2.91%	0.91
heating oil	$y_t^{(1)}$	1 month	-1.27%	4.53%	-30.29%	5.66%	0.46
($N = 155$)	$y_t^{(2)}$		-0.92%	3.48%	-23.43%	3.79%	0.52
	$y_t^{(3)}$		-0.72%	2.76%	-17.89%	3.06%	0.59
	$y_t^{(4)}$		-0.62%	2.32%	-14.64%	2.86%	0.64
live cattle	$y_t^{(1)}$	2 months	-1.78%	4.06%	-10.22%	7.80%	0.33
($N = 102$)	$y_t^{(2)}$		-1.35%	3.00%	-8.34%	5.08%	0.49
	$y_t^{(3)}$		-0.92%	2.17%	-5.33%	3.63%	0.56
	$y_t^{(4)}$		-0.68%	1.69%	-4.04%	3.04%	0.60
soybeans	$y_t^{(1)}$	2 months	0.40%	2.30%	-9.32%	6.89%	0.30
($N = 137$)	$y_t^{(2)}$		0.40%	2.62%	-18.01%	3.89%	0.36
	$y_t^{(3)}$		0.31%	2.74%	-20.59%	3.05%	0.47
	$y_t^{(4)}$		0.29%	2.32%	-15.47%	2.86%	0.58
Deutsche Mark	$y_t^{(1)}$	3 months	0.76%	1.00%	-1.65%	2.96%	0.73
($N = 76$)	$y_t^{(2)}$		0.79%	0.88%	-1.55%	2.43%	0.81
	$y_t^{(3)}$		0.68%	0.83%	-1.48%	2.12%	0.81

term structure of US interest rates is upward sloping, this implies that the risk premia for the convenience yield of soybeans are negative and larger in absolute value than the risk premia in the term structure of interest rates. Finally, for Deutsche Mark futures the average yield curve (which is the yield curve for the interest rate differential between the US and Germany) appears to be flat.

7.4.2 Empirical results for low frequency data

Table 7.2 presents the OLS estimates for the regressions in (7.5). Recall that $\alpha_2 = -\alpha_1$, and $\beta_2 = 1 - \beta_1$. Therefore, Table 7.2 only presents estimates of α_1 and β_1. For three contracts - heating oil, live cattle, and Deutsche Marks - the slope coefficients β_1 are less than one standard deviation away from one. If the coefficient β_1 is indeed one for these futures contracts, this implies that all variation in the current spread $(f_t^{(n)} - f_t^{(1)})$ is due to variation in expected future spreads

Table 7.2: Premium regressions for low frequency data

The table contains estimates for the regressions specified in the top row of the table. Observations are from March 1970, or from the beginning of trading in the contract, until December 1993. Note that one period refers to one month for heating oil contracts, two months for gold, live cattle, and soybean contracts, and three months for Deutsche Mark contracts

$$f_{t+1}^{(n-1)} - s_{t+1} = \alpha_1 + \beta_1(f_t^{(n)} - f_t^{(1)}) + \eta_{1,t+1}$$
$$(s_{t+1} - f_t^{(1)}) - (f_{t+1}^{(n-1)} - f_t^{(n)}) = \alpha_2 + \beta_2(f_t^{(n)} - f_t^{(1)}) + \eta_{2,t+1}$$

Contract	n	α_1 ($-\alpha_2$)	β_1 ($1-\beta_2$)	s.d.(α)	s.d.(β)	R_1^2	R_2^2	D.W.
gold	*2*	0.09%	0.45	0.04	0.03	0.70	0.78	2.00
	3	0.14%	0.70	0.09	0.03	0.82	0.47	1.85
	4	0.18%	0.79	0.15	0.03	0.84	0.28	1.81
heating oil	*2*	-0.73%	1.00	0.30	0.11	0.37	0.00	2.25
	3	-0.95%	1.04	0.42	0.09	0.47	0.00	2.25
	4	-1.03%	1.00	0.45	0.07	0.56	0.00	2.20
live cattle	*2*	-0.84%	1.03	0.27	0.09	0.59	0.00	1.89
	3	-1.71%	1.07	0.40	0.09	0.58	0.01	1.66
	4	-1.90%	1.03	0.43	0.09	0.59	0.00	1.73
soybeans	*2*	-0.30%	0.15	0.19	0.05	0.06	0.66	1.54
	3	-0.58%	0.36	0.40	0.06	0.23	0.48	2.02
	4	-0.42%	0.62	0.57	0.07	0.38	0.17	2.10
Deutsche Mark	*2*	0.07%	1.05	0.09	0.09	0.67	0.01	2.22
	3	0.13%	0.93	0.15	0.07	0.70	0.01	2.09

and not to variation in risk premia. For these contracts then, the expected next period's spread $E_t[(f_{t+1}^{(n-1)} - s_{t+1})]$ only differs from the current spread, $(f_t^{(n)} - f_t^{(1)})$, by a constant α_1.

If $\beta_1 = 1$, the constant α_1 is equal to the risk premium that investors expect to earn by holding an n-period futures contract rather than a 1-period futures contract. This is also the interpretation that follows directly from the second regression in (7.5). The fact that the coefficient β_2 is less than one standard deviation away from zero for the three contracts mentioned above, provides direct evidence for a constant risk premium, which is then equal to $-\alpha_2$. For instance, in buying a 1-month heating oil contract rather than a 2-month contract, an investor expects to earn an extra return of 0.73% per month. The negative risk premia imply that expected returns are always smaller for longer maturity contracts.

For gold and soybean contracts the current spreads contain information about future spreads and about risk premia, since both the estimates of β_1 and of β_2 are significantly different from zero for these contracts. Also, for these contracts, the R^2's are usually rather high for both regressions, while for the other three contracts R_2^2 is always approximately equal to zero. Since for both gold and soybean contracts the estimated β_2 is larger than zero, the expected excess returns on long term contracts over short term contracts is smaller (larger) when the spread between long and short term contracts is large (small).

Alternatively, the results in Table 7.2 can be interpreted in terms of the term structure of interest rates and net cash flow yields. Assuming for instance that for heating oil and live cattle the risk and magnitude of convenience yields is far more important than of interest rates, Table 7.2 provides direct evidence on the term structure of convenience yields. In these terms, the fact that the estimated coefficient β_1 (β_2) is not significantly different from one (zero) means that a version of the expectations hypothesis with constant risk premia can not be rejected for the convenience yields of heating oil and live cattle: expected future convenience yields only differ from forward convenience yields by constant risk premia. Taking the example of heating oil again: the 2-month convenience yield for heating oil equals the average of the current and next period's expected 1-month convenience yields, plus a constant premium of $\frac{1}{2} \times 0.73 = 0.37\%$ per month. This represents a risk premium that is also significant in economic terms. If β_2 is zero, the parameter estimates of α_2 provide a direct estimate of the risk premia $\theta^{(n)}$, implying an upward sloping yield curve for oil.

For Deutsche Marks we can not even reject the hypothesis that the risk premia are zero. Since the spread (yield) consists in this case of the difference of two interest rates, this suggests that the liquidity premium in the term structure of interest rates of the U.S. and Germany are approximately of the same magnitude and cancel out in the futures returns.

As pointed out in Section 7.3.2, if the simple one-factor model for the yields is true, then one implication is that in the regressions in (7.5) differences in maturity, i.e., n, show up in $\alpha_i,\ i = 1, 2$, while β_i is the same for all maturities n. From the results in Table 7.2 it appears that for heating oil, live cattle, and Deutsche Mark futures the estimated β_i are indeed approximately the same for all maturities, which is consistent with a one-factor model. Moreover, for these contracts we can not reject the hypothesis that $\beta_1 = 1$, suggesting that a Vasicek-like model for the yields may be a reasonable model. For gold and soybean futures on the other hand, the fact that β_1 is significantly smaller than one implies that the Vasicek-specification is not valid. If the term structures of yields can be modelled with an affine one-factor model, the short yield must include a heteroskedastic innovation as in the CIR-specification to explain the results in Table 7.2.

Some indirect evidence on the Vasicek- and the CIR-specifications can be obtained by comparing the results of Table 7.1 and Table 7.2. The average slope of the futures term structure can be measured for instance by $E[y_{t+1}^{(2)}] - E[y_{t+1}^{(1)}]$ which for the one-factor model is equal to

$$E[y_{t+1}^{(2)}] - E[y_{t+1}^{(1)}] = 0.5(1-\omega)\sigma_{\varepsilon m} + 0.5\omega\sigma_{\varepsilon m}\mu.$$

Thus, for the Vasicek-specification ($\omega = 0$) the mean slope equals $0.5\sigma_{\varepsilon m}$ and for the CIR-specification ($\omega = 1$) it equals $0.5\sigma_{\varepsilon m}\mu$. Given that $0 \leq \omega \leq 1$, the slope parameter β_1 in (7.10) is smaller (larger) than one if $\omega > 0$ and $\sigma_{\varepsilon m}$ is positive (negative). For instance, in case of gold futures, the estimated slope parameters in Table 7.2 are smaller than one, from which it can be derived that $\sigma_{\varepsilon m} > 0$. Combined with the positive average short term yield for gold, both the Vasicek-specification and the CIR-specification imply an upward sloping average term structure, which is consistent with the mean yields in Table 7.1. However, the fact that the slope coefficient β varies with the maturity is inconsistent with the one-factor model.

For soybeans, both the Vasicek-specification and the CIR-specification of the one-factor model appear to be incapable of explaining the evidence in Table 7.1 and 7.2. As with gold, the estimated slope parameter β_1 implies a positive estimate of the covariance $\sigma_{\varepsilon m}$. The average short yield in Table 7.1 is also positive, implying an upwards sloping average term structure for both the Vasicek and the CIR-like models. Looking at the average yields in Table 7.1 shows that the average yield curve is downward sloping however. Therefore, the evidence for soybean futures shows that the one-factor models have the same kind of difficulty in explaining the term structure of yields as reported by BFMW (1997) for the term structure of interest rates: the one-factor model can not account for both the average slope of the yield curve and the regression coefficient β_1 being smaller than one.[29] The same kind of reasoning shows that for heating oil and Deutsche Mark futures we do not find this inconsistency between the estimated parameters and the slope of the futures term structure, whereas for live cattle the evidence appears to be inconsistent with the Vasicek model but not with the CIR-model.

Direct evidence on whether the Vasicek-model or the CIR-model provides a valid description of the term structure of yields is given in Table 7.3. Recall that an affine one-factor model for the term structure of yields implies that the risk premia are affine functions of the short term yield. From (7.4) and (7.9) we have that:

$$\begin{aligned} \theta_t^{(n)} &= E_t[f_{t+1}^{(n-1)} - s_{t+1}] - (f_t^{(n)} - f_t^{(1)}) \\ &= B^{(n-1)}((1-\omega) + \omega y_t^{(1)})\sigma_{\varepsilon m} \\ &= \gamma^{(n)} + \delta^{(n)} y_t^{(1)}, \end{aligned}$$

which defines $\gamma^{(n)}$ and $\delta^{(n)}$. Based on this, Table 7.3 provides SUR-estimates for the system:

$$\begin{aligned} (f_{t+1}^{(n-1} - s_{t+1}) - (f_t^{(n)} - f_t^{(1)}) &= \gamma^{(n)} + \delta^{(n)} y_t^{(1)} + u_{t+1}^{(n)}, && (7.12a) \\ y_{t+1}^{(1)} &= c + \rho y_t^{(1)} + ((1-\omega) + \omega y_t^{(1)})\varepsilon_{t+1}, && (7.12b) \end{aligned}$$

29 Although BFMW consider a slightly different regression, the problem they encounter with the one-factor models for interest rates is the same as the problem we describe here for the term structure of soybean futures.

Table 7.3: Test for the Vasicek and CIR models

The table contains SUR estimates for the regressions specified in the top row of the table. Vasicek is a Wald test for the restrictions imposed on the intercepts and slope coefficients by the Vasicek model, i.e. for $\omega = 0$. CIR is a Wald test for the restrictions imposed on the intercepts and slope coefficients by the CIR model, i.e. for $\omega = 1$. These test statistics are χ^2_5 except for the Deutsche Mark, where they are χ^2_3. The line '(slopes only)' presents the test statistics for the restrictions imposed by the Vasicek and CIR models on the slope coefficients only. For the Vasicek-like model these statistics are χ^2_3 and for the CIR-like model χ^2_2, except for the Deutsche Mark futures, where they are χ^2_2 and χ^2_1 resp. Observations are from March 1970, or from the beginning of trading in the contract, until December 1993. Note that one period refers to one month for heating oil contracts, two months for gold, live cattle, and soybean contracts, and three months for Deutsche Mark contracts.

$$(f_{t+1}^{(n-1)} - s_{t+1}) - (f_t^{(n)} - f_t^{(1)}) = \gamma^{(n)} + \delta^{(n)} * y_t^{(1)} + u_{t+1}^{(n)}$$
$$y_{t+1}^{(1)} = c + \rho * y_t^{(1)} + \varepsilon_{t+1}$$

gold	n	$\gamma^{(n)}$	s.d.($\gamma^{(n)}$)	$\delta^{(n)}$	s.d.($\delta^{(n)}$)	R^2
	2	0.01%	0.05	-0.94	0.07	0.65
	3	0.01%	0.10	-0.98	0.12	0.36
	4	0.00%	0.15	-0.96	0.19	0.20
		c	s.d.(c)	ρ	s.d.(ρ)	
		0.17%	0.05	0.75	0.06	0.57
		Vasicek:	900.30**	*CIR:*	1169.70**	
		(slopes only)	280.23**		203.96**	
heating oil	n	$\gamma^{(n)}$	s.d.($\gamma^{(n)}$)	$\delta^{(n)}$	s.d.($\delta^{(n)}$)	R^2
	2	-0.76%	0.30	-0.03	0.07	0.00
	3	-1.00%	0.43	-0.01	0.09	0.00
	4	-1.06%	0.46	-0.02	0.10	0.00
		c	s.d.(c)	ρ	s.d.(ρ)	
		-0.69%	0.34	0.46	0.07	0.22
		Vasicek:	10.82	*CIR:*	15.44**	
		(slopes only)	9.40*		4.00	
live cattle	n	$\gamma^{(n)}$	s.d.($\gamma^{(n)}$)	$\delta^{(n)}$	s.d.($\delta^{(n)}$)	R^2
	2	-0.83%	0.28	0.02	0.07	0.01
	3	-1.52%	0.42	0.14	0.10	0.02
	4	-1.65%	0.45	0.16	0.10	0.02
		c	s.d.(c)	ρ	s.d.(ρ)	
		-1.17%	0.42	0.33	0.10	0.11
		Vasicek:	18.80**	*CIR:*	29.71**	
		(slopes only)	6.90		6.82	

for $n = 1, 2, ..., K$, and where we choose $\omega = 0$ or $\omega = 1$, i.e. a Vasicek or a CIR-like model. If ω is either one or zero, we know the exact relationship between the coefficients $\gamma^{(n)}$, $\delta^{(n)}$, and ρ. These relationships impose nonlinear restrictions on the coefficients in (7.12). Table 7.3 also presents Wald-tests for the nonlinear restrictions imposed by the hypothesis that $\omega = 0$ (labelled *Vasicek*) and by the hypothesis that $\omega = 1$ (labelled *CIR*). The reported test-statistics are for the restrictions imposed by the two specifications on both the intercepts and the slope coefficients as

Table 7.3: Test for the Vasicek and CIR models (continued)

soybeans	n	$\gamma^{(n)}$	s.d.$(\gamma^{(n)})$	$\delta^{(n)}$	s.d.$(\delta^{(n)})$	R^2
	2	0.16%	0.32	-0.52	0.14	0.09
	3	0.49%	0.55	-0.62	0.24	0.05
	4	0.27%	0.63	-0.34	0.27	0.01
		c	s.d.(c)	ρ	s.d.(ρ)	
		0.24%	0.19	0.30	0.08	0.08
		Vasicek:	47.65**	*CIR:*	25.91**	
		(slopes only)	42.88**		21.25**	
Deutsche Mark	n	$\gamma^{(n)}$	s.d.$(\gamma^{(n)})$	$\delta^{(n)}$	s.d.$(\delta^{(n)})$	R^2
	2	0.07%	0.09	0.05	0.07	0.01
	3	-0.05%	0.15	0.11	0.12	0.01
		c	s.d.(c)	ρ	s.d.(ρ)	
		0.15%	0.10	0.77	0.08	0.57
		Vasicek:	0.88	*CIR:*	3.73	
		(slopes only)	0.88		0.82	

well as on the slope coefficients only. Since under the hypothesis that $\omega = 1$ the AR(1) process for the short yield in (7.12) is heteroskedastic, the Wald test for this hypothesis is based on GLS-estimates of the system in (7.12). All other reported results in Table 7.3 are for the homoskedastic case however.

As in Table 7.2, the results in Table 7.3 show first of all that the risk premia of heating oil, live cattle, and Deutsche Mark futures do not depend on the short yield, $y_t^{(1)}$, while for gold and soybean futures they do. For heating oil and live cattle futures the risk premia are constant and significantly different from zero, while for Deutsche Mark futures, we are again not able to reject the hypothesis that risk premia are zero. The Wald test rejects the hypothesis that the Vasicek-like model ($\omega = 0$) provides a good specification of the term structure of yields for gold and soybean futures, as well as for live cattle futures, but not for heating oil and Deutsche Mark futures. If we only test the restrictions imposed on the slope coefficients however, the Vasicek-like model can be rejected for the heating oil futures as well. A formal test shows that the CIR-like model ($\omega = 1$) is rejected for all contracts except the Deutsche Mark futures. The fact that we can not reject the two models for the Deutsche Mark futures however, is due to the fact that the risk premia for Deutsche Mark futures are essentially zero. This latter conclusion is consistent with the results of e.g. Hakkio & Leiderman (1986) who can not reject the joint hypothesis of uncovered interest parity and the expectations hypothesis for monthly data. McCurdy & Morgan (1987) on the other hand, do reject this joint hypothesis using weekly data.

Notice that our CIR-model for the Deutsche Mark futures yields is similar to the affine models with interdependent factors for currency pricing that are considered by Backus, Foresi & Telmer (BFT) (1997). In these models interest rates in two countries are affected by common factors in an asymmetric way. BFT show that such a model provides a better fit of the data than a model where the interest rates in the two countries are affected by independent factors. They provide estimates for a model with two interdependent factors, because their purpose is to model the interest rates in both countries as well as the exchange rate, rather than just the interest rate differential as we do here.

Summarizing, except for heating oil and Deutsche Mark futures, neither the Vasicek-like model, nor the CIR-model provides a good specification of the term structure of yields for the futures contracts considered here. Given the failure of the one-factor model to explain the regression evidence and the average slope of the term structure for gold, soybeans and live cattle, which (for soybeans) is similar to the problem reported by BFMW (1997) for one-factor models of the term structure of interest rates, we probably need two-factor models to characterize the futures term structure for these commodities.

7.4.3 Empirical results for daily data

As pointed out above, if we base our analysis on the regressions in (7.5) then we need to observe $y_t^{(1)}$ every period, implying that the frequency of the observations can not exceed the frequency of the delivery dates. A drawback of using those low frequency data is that much information is lost because only a limited number of the observations can be used. On the other hand, using daily data the condition that there are observations for $y_t^{(1)}$ every period is clearly not fulfilled, since futures contracts expire at most once per month. A similar analysis as in the preceding section can be performed for daily data as well however, if we start from the decomposition of $h_t^{(k,n)}$ rather than from the decomposition of $h_t^{(1,n)}$:

$$\begin{aligned}(n-k)h_t^{(k,n)} &= ny_t^{(n)} - ky_t^{(k)} \\ &= E_t[(n-k)y_{t+k}^{(n-k)}] + \Theta_t^{(k,n)}, \\ \Theta_t^{(n,k)} &\equiv \sum_{i=0}^{k-1} \theta_t^{(n-i)}.\end{aligned} \tag{7.13}$$

Equation (7.13) is a straightforward generalization of the decomposition in (7.3). Again, it is convenient to express (7.13) in terms of spreads between futures and spot prices rather than in terms of yields. This gives the following generalization of the regressions in (7.5):

$$f_{t+k}^{(n-k)} - s_{t+k} = \alpha_1 + \beta_1(f_t^{(n)} - f_t^{(k)}) + \eta_{1,t+k} \tag{7.14a}$$

$$(s_{t+k} - f_t^{(k)}) - (f_{t+k}^{(n-k)} - f_t^{(n)}) \;=\; \alpha_2 + \beta_2(f_t^{(n)} - f_t^{(k)}) + \eta_{2,t+k}. \qquad (7.14b)$$

Obviously, the interpretation of (7.14) is completely analogous to the interpretation of (7.5). The first regression in (7.14) answers the question whether the current forward spread between $f_t^{(n)}$ and $f_t^{(k)}$ has predictive power for the $(n-k)$-period spread k periods ahead. If this is the case, then β_1 will be different from zero. The second regression investigates whether there is variation in the k-period risk premium $\Theta_t^{(k,n)}$ that shows up in the current spread. Note that the left-hand-side of the second regression is the return on a spreading strategy that involves a long position in a k-period futures contract and a short position in an n-period futures contract, and holding these positions for k periods. The expected return on this strategy equals $-\Theta_t^{(k,n)}$.

Table 7.4: Premium regressions for daily data

The table contains estimates for the regressions specified in the top row of the table. The first column indicates that i-th and j-th nearest-to-maturtiy contracts are used. Observations are for the period March 1970 until December 1993.For all contracts 3000 observations are available, except for heating oil, for which respectively 2626, 2442, and 2008 observations are available. Reported standard errors are Newey-West standard errors.

$$f_{t+k}^{(n-k)} - s_{t+k} = \alpha_1 + \beta_1(f_t^{(n)} - f_t^{(k)}) + \eta_{1,t+k}$$
$$(s_{t+k} - f_t^{(k)}) - (f_{t+k}^{(n-k)} - f_t^{(n)}) = \alpha_2 + \beta_2(f_t^{(n)} - f_t^{(k)}) + \eta_{2,t+k}$$

Contract		α_1 $(-\alpha_2)$	β_1 $(1-\beta_2)$	s.d.[α]	s.d.[β]	R_1^2	R_2^2
gold	*1, 2*	0.18%	0.28	0.17	0.13	0.02	0.11
	1, 3	0.20%	0.64	0.19	0.07	0.26	0.10
	1, 4	0.24%	0.76	0.19	0.04	0.56	0.12
heating oil	*1, 2*	-2.31%	1.48	0.87	0.20	0.13	0.02
	1, 3	0.09%	1.06	0.31	0.11	0.66	0.01
	1, 4	-0.56%	1.20	0.51	0.14	0.70	0.06
live cattle	*1, 2*	1.47%	1.31	0.20	0.06	0.77	0.16
	1, 3	0.96%	1.16	0.39	0.06	0.75	0.06
	1, 4	0.68%	1.05	0.45	0.05	0.75	0.01
soybeans	*1, 2*	1.56%	0.62	0.21	0.09	0.36	0.17
	1, 3	1.49%	0.81	0.37	0.09	0.59	0.07
	1, 4	1.37%	0.93	0.59	0.12	0.66	0.01
Deutsche Mark	*1, 2*	0.04%	0.96	0.07	0.07	0.80	0.01
	1, 3	0.02%	1.00	0.09	0.05	0.90	0.00

Table 7.4 presents estimates of the regressions in (7.14) for daily observations of the futures contracts. Note that n and k are measured in days now. Each regression is based on daily observations of a pair of contracts, where the first column in Table 7.4 indicates which contracts are used. For instance, *1, 2* means that the nearest-to-maturity and the second nearest-to-maturity contracts are used. Since delivery dates are fixed, if at day t we observe contracts with n and k days to maturity, then at day $t+1$ we observe contracts with $n-1$ and $k-1$ days to maturity

(unless day t is a delivery day). This implies that the observations are overlapping for at most $n-k$ days. Therefore, the standard errors in Table 7.4 are calculated as in Newey & West (1987a).

As with the low frequency observations, the estimates of β_1 and β_2 for heating oil and Deutsche Mark futures show that it is mainly variation in the future spreads that shows up in the forward spreads, while the forward spreads do not contain much information about risk premia, i.e. about expected holding returns on spreading strategies. The only exception with regard to the oil contracts are the spreads between the nearest-to-maturity and second nearest-to-maturity oil contracts, where the estimate of β_2 is significantly different from zero, indicating that the forward spread does contain information about risk premia. The results for the Deutsche Mark futures are especially close to the results of the low frequency data in Table 7.2, showing intercepts close to zero and slope coefficients β_1 (β_2) close to one (zero).

The estimates for live cattle in Table 7.4 are different from the low frequency results in that variation in risk premia now shows up in variation in the forward spreads, except for the longest-to-maturity spread. This can be seen from the estimates of β_2 which are significantly different from zero. For the daily data the results for live cattle futures are now similar to the results for gold and soybean futures. For these three contracts the risk premia depend on the slope of the current futures term structure.

Again, the results in Table 7.4 can be interpreted in terms of spreads and returns on spreading strategies, as well as in terms of the term structure of yields. For instance, the estimates for Deutsche Mark futures again suggest that the expectations hypothesis for the yields (i.e., for the interest *differential*) can not be rejected and that risk premia are zero. Although this is consistent with the low frequency results in the previous section as well as with the results for monthly observations in Hakkio & Leiderman (1986), these findings do not confirm the results of McCurdy & Morgan (1987) for weekly observations.

Similar to Section 7.3.2, we can express the regression coefficients in (7.14) in terms of the parameters of the affine one-factor model. Specifically, the one-factor model implies that the slope coefficients can be written as:

$$\begin{aligned} \beta_1 &= \frac{\rho^k}{(\rho+\omega\sigma_{\varepsilon m})^k}, \\ \beta_2 &= 1-\frac{\rho^k}{(\rho+\omega\sigma_{\varepsilon m})^k}, \end{aligned} \tag{7.15}$$

and that the constant α_1 ($=-\alpha_2$) equals:

$$\begin{aligned} \alpha_1 &= \Psi\left\{\frac{\varphi-\varphi^{n-k}}{1-\varphi}-\rho^k\frac{1-\varphi^{n-k}}{1-\varphi}\right\}, \\ \text{with } \Psi &\equiv (1-\rho)\mu+(1-\omega)\sigma_{\varepsilon m}, \end{aligned} \tag{7.16}$$

$$\varphi \equiv \rho + \omega\sigma_{\varepsilon m}.$$

If $\omega = 0$ and $\rho \neq 0$, i.e., if the process for the short term yield $y_t^{(1)}$ is homoskedastic and the short yield does not immediately revert to its long term average, then we obtain again that $\beta_1 = 1$ and that $\beta_2 = 0$. Similarly, the opposite extreme case in which $\beta_1 = 0$ and $\beta_2 = 1$ is obtained when $\rho = 0$ and $\omega\sigma_{\varepsilon m} \neq 0$, as before. More importantly, note from (7.15) and (7.16) that both the intercepts and the slope coefficients are of the form $f \pm g^k$, with $g \geq 0$. The $\pm$-sign denotes an indicator variable that is either $+1$ or -1. It is computationally convenient to impose the condition that g is nonnegative. Also, this condition prevents the coefficients (and the risk premia) to show a switching pattern when the number of days to maturity is either odd or even. Table 7.5 reports estimates of the regressions in (7.14) in which the parameters are functions of k, the maturity of the nearest-to-maturity contract:

$$\begin{aligned} f_{t+k}^{(n-k)} - s_{t+k} &= (a_1 \pm c_1^k) + (b_1 \pm d_1^k)(f_t^{(n)} - f_t^{(k)}) + \eta_{1,t+k} \\ (s_{t+k} - f_t^{(k)}) - (f_{t+k}^{(n-k)} - f_t^{(n)}) &= (a_2 \pm c_2^k) + (b_2 \pm d_2^k)(f_t^{(n)} - f_t^{(k)}) + \eta_{2,t+k}, \end{aligned}$$

where we impose that $c_i > 0$ and $d_i > 0$, $i = 1, 2$. Note that these regressions are again complementary in that $a_1 = -a_2$, $c_1^k = -c_2^k$, $b_1 = 1 - b_2$, and $d_1^k = -d_2^k$.

According to equation (7.15) b_1 should be equal to zero and d_1 should be equal to the estimated slope coefficient β_1 for the low frequency results in Table 7.3, unless the Vasicek model is true, i.e., $\omega = 0$, in which case it is also possible that $b_1 = 1$ and $d_1 = 0$. The hypothesis that $b_1 = 0$ can almost always be rejected. For heating oil and Deutsche Mark futures however, the hypothesis that $b_1 = 1$ and that $d_1 = 0$ can not be rejected, which is in accordance with a Vasicek-like model for these contracts. For most other contracts both the hypothesis that $b_1 = 0$ and that $b_1 = 1$ are rejected and there the evidence contradicts the one-factor model. Finally, the intercept for Deutsche Mark futures appears to be equal to zero again, lending more support to the expectations hypothesis with zero risk premia for the yields on these contracts.

7.5 Summary and conclusions

This chapter analyzes differences in one-period risk premia for futures contracts with different maturities. These differences are caused by risk premia in the term structure of yields, where the yield is defined as the annualized spread between the futures and the spot price, which is determined by interest rates, dividend yields, convenience yields, storage costs, etc. Our analysis focuses on the information in the current term structure of futures prices (yields) about expected future spreads (yields) and risk premia therein. Using a simple affine one-factor model for the term structure of yields, that has a Vasicek and CIR-like model as special cases, more precise statements about the

Table 7.5: Premium regressions for daily daya with time-varying parameters

The table contains estimates for the regressions specified in the top row of the table. The first column indicates that i-th and j-th nearest-to-maturtiy contracts are used. Observations are for the period March 1970 until December 1993. For all contracts 3000 observations are available, except for heating oil, for which respectively 2626, 2442, and 2008 observations are available. A minus sign for c_1 and d_1 means that the time intercept and the slope coefficients should be read as $a_1 - c_1^k$ and $b_1 - d_1^k$ respectively, where c_1 and d_1 themselves are always positive. Reported standard errors are Newey-West standard errors.

$$f_{t+k}^{(n-k)} - s_{t+k} = (a_1 \pm c_1^k) + (b_1 \pm d_1^k) * (f_t^{(n)} - f_t^{(k)}) + \eta_{1,t+k}$$
$$(s_{t+k} - f_t^{(k)}) - (f_{t+k}^{(n-k)} - f_t^{(n)}) = (a_2 \pm c_2^k) + (b_2 \pm d_2^k) * (f_t^{(n)} - f_t^{(k)}) + \eta_{2,t+k}$$

Contract		a_1	c_1	b_1	d_1	R_1^2	R_2^2
gold	*1,2*	0.15%	-0.25%	0.29	0.56	0.02	0.11
		[0.18]	[0.04]	[0.13]	[0.15]		
	1,3	0.18%	-0.58%	0.07	0.46	0.27	0.10
		[0.20]	[0.06]	[0.07]	[0.05]		
	1,4	0.24%	-0.82%	0.75	0.37	0.57	0.12
		[0.19]	[0.10]	[0.04]	[0.03]		
heating oil	*1,2*	-2.34%	2.34%	1.56	-0.46	0.13	0.02
		[0.87]	[0.84]	[0.32]	[0.41]		
	1,3	0.05%	0.40%	1.04	0.33	0.66	0.01
		[0.31]	[0.35]	[0.12]	[0.18]		
	1,4	-0.59%	0.84%	1.21	-0.05	0.70	0.06
		[0.53]	[0.49]	[0.15]	[0.16]		
live cattle	*1,2*	1.45%	0.81%	1.31	-0.13	0.77	0.16
		[0.20]	[0.22]	[0.06]	[0.08]		
	1,3	0.94%	1.04%	1.17	-0.12	0.75	0.06
		[0.39]	[0.37]	[0.06]	[0.07]		
	1,4	0.65%	1.31%	1.05	-0.01	0.75	0.01
		[0.46]	[0.45]	[0.05]	[0.06]		
soybeans	*1,2*	1.55%	0.39%	0.63	0.02	0.36	0.17
		[0.21]	[0.16]	[0.09]	[0.09]		
	1,3	1.47%	0.52%	0.81	0.03	0.59	0.07
		[0.38]	[0.25]	[0.09]	[0.08]		
	1,4	1.36%	0.61%	0.93	-0.02	0.66	0.01
		[0.60]	[0.38]	[0.12]	[0.09]		
Deutsche Mark	*1,2*	0.04%	- -	0.96	-0.04	0.80	0.01
		[0.07]	- -	[0.07]	[0.05]		
	1,3	0.02%	0.05%	1.00	-0.02	0.90	0.00
		[0.09]	[0.06]	[0.05]	[0.03]		

information in the term structure of futures prices can be made. We show that it is only in the Vasicek specification of the term structure that risk premia are constant and that the futures term structure does not contain any information about risk premia.

The empirical analysis shows that the Vasicek model can not be rejected for heating oil and Deutsche Mark futures contracts. If the Vasicek model is valid, it is relatively straightforward to derive the covariances of the pricing kernel and all yields from the term structure of futures prices. For heating oil we find evidence that risk premia are constant and negative, implying that expected one-period returns are always higher for the short maturity contracts and that the term structure of yields is upward sloping. Of course this has clear implications for hedging and portfolio decisions. For Deutsche Mark futures we can not reject the hypothesis that risk premia are zero. Since the yield for Deutsche Mark futures is the differential between the German and U.S.-interest rates, this means that for this interest *differential* we can not reject the expectations hypothesis with zero risk liquidity premia.

For gold and soybean futures we find evidence that the expected one period futures returns depend on the slope of the futures term structure, where the expected return on long term contracts relative to short term contracts is smaller (larger) when the spread between the long and short term contracts is larger (smaller), i.e. when the term structure is more upward sloping. Finally, for live cattle futures the evidence is mixed. For these latter three contracts the one-factor model is unable to explain the regression evidence and the average slope of the yield curve. The variation in the risk premia can not be captured by a simple one-factor model such as the CIR-model. This suggests that in future research it may be useful to model the term structure of yields with a multi-factor model.

Chapter 8

Summary

This thesis consists of two parts. Part I is about spanning and intersection. The central question in this part is whether or not investors that invest in a set of benchmark assets can extend their efficient set by investing in a additional set of assets as well. In terms of mean-variance frontiers, if the mean-variance frontier for the benchmark assets and the mean-variance frontier for the benchmark assets plus the additional asset have exactly one point in common, there is mean-variance intersection. If these two mean-variance frontiers coincide, there is mean-variance spanning.

Chapter 2 provides an introduction to the concept of mean-variance spanning and intersection and its relation to various parts of the literature. Because intersection and spanning have an interpretation in terms of mean-variance frontiers as well as volatility bounds, we started by illustrating the duality between these two frontiers. It is shown that, except for the global minimum variance portfolio, each portfolio on the mean-variance frontier has a unique corresponding point on the volatility bound of stochastic discount factors. When deriving the restrictions that mean-variance intersection and spanning imply for the distribution of the benchmark assets and the additional assets, we can start both from the mean-variance frontier and from the volatility bound. Intersection and spanning can be defined in terms of both frontiers and imply identical restrictions on the distribution of the assets returns.

Tests for intersection and spanning have a natural interpretation in terms of Jensen's alpha and the Sharpe ratio. Knowing Jensen's alpha allows one to determine the potential improvement in Sharpe ratios, and the potential improvement in (sample) Sharpe ratios determines the intersection and spanning test statistics. Therefore, the regressions and test statistics that are used to test for intersection and spanning also have a clear economic interpretation. Moreover, the regression estimates together with the initial portfolio characteristics, allow one to determine the new optimal portfolio weights as well. Finally, there appears to be a close relation between intersection and the specification error bounds introduced by Hansen & Jagannathan (1997). When using the minimum variance stochastic discount factor for a subset of assets as a proxy for a larger set of assets, specification error bounds can be used as a measure for the deviation from intersection.

In Chapter 3 we generalize the notion of mean-variance spanning as described in Chapter 2 in three dimensions. First of all we show how regression techniques can be used to test for spanning for more general classes of utility functions. It is shown that in projecting a security's return on a specific class of kernels and portfolios of the initial set of securities, spanning implies that the projection yields a portfolio of these securities. Second, the tests for spanning are generalized to the case of zero-investment securities like futures, forwards, and swaps. If zero-investment securities are considered, then spanning implies restrictions on the coefficients in the spanning regression that reflect the zero-investment property. Finally, we show how to test for spanning in case there are nontraded assets. If an investor has a position in a nontraded asset, then this changes his investment opportunity set. In spanning tests this can be incorporated by using returns that are adjusted for the return on the nontraded asset.

We test whether three international stock indices, i.e., the S&P 500, the FAZ (Germany), and the FTSE (UK), span a set of commodity futures and currency futures. If it is assumed that all relevant moments of monthly holding returns are constant, and that there are no market frictions like short selling constraints and transaction costs, we can reject the hypothesis that there is spanning for most futures contracts, but whether or not the spanning hypothesis is rejected depends on the specific utility functions of interest. If an investor has a nonmarketable position in a commodity underlying one of the futures contracts, the hypothesis of spanning can almost always be rejected for the futures contract on that same commodity for all utility functions considered. Moreover, a nonmarketable position in one agricultural commodity usually implies that the hypothesis of spanning is rejected for most of the agricultural futures contracts. If there is an exposure to a foreign currency, spanning can only be rejected for investors with power utility functions that reflect a preference for skewness. Finally, allowing expected returns to depend on the net positions of large hedgers in the futures market, spanning can be rejected for many futures contracts for all utility functions considered.

In Chapters 2 and 3 we assume that there are no market frictions such as short sales constraints and transaction costs. In Chapter 4 we show how regression techniques can be used to test for mean-variance spanning and intersection in case there are short sales constraints and/or transaction costs. When there are short sales constraints on the benchmark assets, the mean-variance frontier consists of parts of the mean-variance frontiers of subsets of the set of benchmark assets. If the benchmark assets are to span a new set of assets, there has to be spanning for each subset of the benchmark assets. This can be incorporated in regression based tests for spanning, by using a multivariate regression in which the returns on the new assets are regressed on the returns of the relevant subsets of the benchmark assets. Short sales restrictions on the new assets require to test for inequality restrictions rather than equalities. Following the ideas presented for instance in Luttmer (1996), transaction costs can be handled by looking at short and long positions in an asset as two different securities. Transaction costs can then be dealt with in the same way as short sales constraints.

Using standard mean-variance spanning tests as described in Chapter 2, the hypothesis that a set of emerging stock market indices are spanned by the MSCI Indices for the US, Europe and Japan, is easily rejected. The hypothesis of spanning can still be rejected for many emerging markets if there are short sales constraints on both the emerging markets and the benchmark assets. However, the markets for which spanning can be rejected when there are short sales constraints, appear to be markets for which foreign ownership restrictions may be particularly severe. Taking into account transaction costs, the evidence against spanning is much weaker, although there is still evidence in favor of the diversification benefits of emerging markets for holding periods of two months or longer. When we estimate the minimum amount of transaction costs that are needed in order not to reject the hypothesis of spanning with monthly trading, this lower bound on the transaction costs is lower than estimates of the actual transactions in all but two markets. Therefore, the analysis in this chapter suggests that in determining the potential diversification benefits of emerging markets for US-investors, it is important to take into account real-life market frictions such as short sales constraints and transaction costs. However, there is still evidence in favor of these diversification benefits, even after allowing for short sales constraints and transaction costs. Finally, if we limit the analysis to the subperiods after some major liberalizations in the emerging stock markets have taken place and take legal ownership restrictions into account, there is little evidence against the hypothesis of spanning, even if there are no market frictions.

Part II of this thesis is about modeling risk premia in futures markets. In Chapter 6 we analyze a simple multifactor factor model for futures risk premia in which risk premia are determined by the covariance of futures returns with the market return, as well as by hedging pressure variables. The model identifies hedging pressure variables from the own futures market as well as from other related futures markets, as the relevant state variables. Hedging pressures from other markets are referred to as cross hedging pressures. We analyze how well the multifactor model, as well as the CAPM, perform for a set of 20 futures contracts that can be grouped into four categories: financial, agricultural, mineral and currency futures. Our specification of the multifactor model uses hedging pressure variables from within each futures own group as the relevant state variables.

We show that hedging pressure variables have a significant effect on futures returns, after controlling for market risk, which is consistent with a multifactor model. In terms of the measure for model misspecification introduced by Hansen & Jagannathan (1997) the model with hedging pressure variables shows a substantial decrease in the specification error bounds relative to a model without hedging pressure. Therefore, the mispricing of futures portfolios can be significantly reduced if cross hedging pressure variables are taken into account besides market risk.

Finally, Chapter 7 analyzes differences in one-period risk premia for futures contracts with different maturities. These differences can be attributed to risk premia in the term structure of yields, where the yield is defined as the annualized spread between the futures and the spot price, which is determined by interest rates, dividend yields, convenience yields, storage costs, etc.

Our analysis focuses on the information in the current term structure of futures prices (yields) about expected future spreads (yields) and risk premia therein. Using a simple affine one-factor model for the term structure of yields, that has a Vasicek and CIR-like model as special cases, more precise statements about the information in the term structure of futures prices can be made. We show that it is only in the Vasicek specification of the term structure that risk premia are constant and that the futures term structure does not contain any information about risk premia.

The empirical analysis shows that the Vasicek model can not be rejected for heating oil and Deutsche Mark futures contracts. For gold and soybean futures we find evidence that the expected one period futures returns depend on the slope of the futures term structure, where the expected return on long term contracts relative to short term contracts is smaller (larger) when the spread between the long and short term contracts is larger (smaller), i.e. when the term structure is more upward sloping. Finally, for live cattle futures the evidence is mixed. Although for these latter three contracts the risk premia depend on the slope of the futures term structure, the variation in the risk premia can not be captured by a simple one-factor model such as the CIR-model.

References

Adler, M., and Dumas, B., 1983, "International Portfolio Choices and Corporation Finance: A Synthesis", *Journal of Financ* e, 38, p.925-984.

Anderson, R.W., and Danthine, J.P., 1981, "Cross Hedging", *Journal of Political Economy*, 89, p.1182-1196.

Backus, D., S. Foresi, A. Mozumdar, and L. Wu, 1997, "Predictable Changes in Yields and Forward Rates", Working Paper, Stern School of Business, New York.

Backus, D., S. Foresi, and C. Telmer, 1997, "Affine Models of Currency Pricing", Working Paper, Stern School of Business, New York.

Bekaert, G., 1995, "Market Integration and Investment Barriers in Emerging Equity Markets", *The World Bank Economic Review*, 9, p.75-107.

Bekaert, G., Erb, C.B., Harvey, C.R., and Viskanta, T.E., 1996, "The Behavior of Emerging Market Returns", CIBER Working Paper 96-002.

Bekaert, G., and Urias, M.S., 1996, "Diversification, Integration, and Emerging Market Closed-End Funds", *Journal of Finance*, 51, p.835-870.

Bessembinder, H., 1992, "Systematic Risk, Hedging Pressure, and Risk Premiums in Futures Markets", *Review of Financial Studies*, 5, p.637-667.

Bessembinder, H., Coughenour, J.F., Seguin, P.J., and Smoller, M.M., 1995, "Mean Reversion in Equilibrium Asset Prices: Evidence from the Futures Term Structure", *Journal of Finance*, 50, p.361-375.

Bessembinder, H., J.F. Coughenour, P.J. Seguin, and M.M. Smoller, 1996, "Is There a Term Structure of Futures Volatilities? Reevaluating the Samuelson Hypothesis", *Journal of Derivatives*, p.45-58.

Black, F., 1976, "The Pricing of Commodity Contracts", *Journal of Financial Economics*, 3, p.167-179.

Brennan, M.J., 1991, "The Price of Convenience and the Valuation of Commodity Contingent Claims", in: Lund, D., and Øksendal, B. (eds.), *Stochastic Models and Option Values*, Elsevier, North Holland.

Campbell, J.Y., Lo, A.W., and MacKinlay, A.C., 1997, *The Econometrics of Financial Markets*, Princeton University Press, Princeton.

Carr, P.P., and Jarrow, R.A., 1995, "A Discrete Time Synthesis of Derivative Security Valuation using a Term Structure of Futures Prices", in: Jarrow, R.A., Maksimovic, V., and Ziemba, W.T. (eds.), *Handbooks in Operations Research and Management Science 9: Finance*, Elsevier, North Holland.

Carter, C.A., Rausser, G.C., and Schmitz, A., 1983, "Efficient Asset Portfolios and the Theory of Normal Backwardation", *Journal of Political Economy*, 91, p.319-331.

Chang, E.C., 1985, "Returns to Speculators and the Theory of Normal Backwardation", *Journal of Finance*, 40, p.193-208.

Chen, Z., and Knez, P.J., 1996, "Portfolio Performance Measurement: Theory and Applications", *Review of Financial Studies*, 9, p.511-556.

Cochrane, J.H., 1997, *Asset Pricing*, Unpublished Manuscript, University of Chicago.

Cox, J.C., Ingersoll, J.E., and Ross, S.A., 1981, "The Relationship between Forward Prices and Futures Prices", *Journal of Financial Economics*, 9, p.321-346.

Cox, J.C., Ingersoll, J.E., and Ross, S.A., 1985, "A Theory of the Term Structure of Interest Rates", *Econometrica*, 53, p.385-407.

Cumby, R.E., and Glen, J.D., 1990, "Evaluating the Performance of International Mutual Funds", *Journal of Finance*, 45, p.497-521.

DeJong, A., DeRoon, F.A., and Veld, C., 1997, "Out-of-Sample Hedging Effectiveness of Currency Futures for Different Models and Hedging Strategies", *Journal of Futures Markets*, 17, p.817-837.

DeRoon, F.A., Nijman, Th.E., and Veld, C., 1997a, "Analyzing Specification Errors in Models for Futures Risk Premia with Hedging Pressure", Working Paper, Tilburg University.

DeRoon, F.A., Nijman, Th.E., and Veld, C., 1997b, "Pricing Term Structure Risk in Futures Markets", *Journal of Financial and Quantitative Analysis*, Forthcoming.

DeRoon, F.A., Nijman, Th.E., and Werker, B.J.M., 1997a, "Testing for Spanning with Futures Contracts and Nontraded Assets: A General Approach", Working Paper, Tilburg University.

DeRoon, F.A., Nijman, Th.E., and Werker, B.J.M., 1997b, "Testing for Mean-Variance Spanning with Short Sales Constraints and Transaction Costs: The Case of Emerging Markets", Working Paper, Tilburg University.

DeSantis, G., 1994, "Asset Pricing and Portfolio Diversification: Evidence from Emerging Financial ", in Howell, M, (ed.): *Investing in Emerging Markets*, Euromoney Books, London.

DeSantis, G., 1995, "Volatility bounds for Stochastic Discount Factors: Tests and Implications from International Financial Markets", Working Paper, University of Southern California.

Duffie, D., 1988, *Dynamic Asset Pricing Theory*, Princeton University Press, Princeton.

Dusak, K., 1973, "Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums", *Journal of Political Economy*, 81, p.1387-1406.

Fama, E.F., 1984a, "Forward and Spot Exchange Rates", *Journal of Monetary Economics*, 14, p.319-338.

Fama, E.F., 1984b, "The Information in the Term Structure", *Journal of Financial Economics*, 13, p.509-528.

Fama, E.F., 1984c, "Term Premiums in Bond Returns", *Journal of Financial Economics*, 13, p.529-546.

Fama, E.F., 1986, "Term Premiums and Default Premiums in Money Markets", *Journal of Financial Economics*, 17, p.175-196.

Fama, E.F., 1996, "Multifactor Portfolio Efficiency and Multifactor Asset Pricing", *Journal of Financial and Quantitative Analysis*, 31, p.441-465.

Fama, E.F., and French, K.R., 1987, "Commodity Futures Prices: Some Evidence on Forecast Power, Premiums, and the Theory of Storage", *Journal of Business*, 60, p.55-73.

Fama, E.F., and French, K.R., 1988, "Business Cycles and the Behavior of Metals Prices", *Journal of Finance*, 43, p.1075-1093.

Fama, E.F., and French, K.R., 1996, "Multifactor Explanations of Asset Pricing Anomalies", *Journal of Finance*, 51, p.55-84.

Ferson, W.E., 1995, "Theory and Empirical Testing of Asset Pricing Models", in: Jarrow, R.A., Maksimovic, V., and Ziemba, W.T. (eds.), *Handbooks in Operations Research and Management Science 9: Finance*, Elsevier, North Holland.

Ferson, W.E., Foerster, S.R., and Keim, D.B., 1993, "General Tests of Latent Variable Models and Mean-Variance Spanning", *Journal of Finance*, 48, p.131-156.

Ferson, W.E., and Schadt, R.W., 1996, "Measuring Funds Strategy and Performance in Changing Economic Conditions", *Journal of Finance*, 51, p.425-462.

Gibbons, M., Ross, S., and Shanken, J., 1989, "A Test of the Efficiency of a Given Portfolio", *Econometrica*, 57, p.1121-1152.

Glen, J., and Jorion Ph., 1993, "Currency Hedging for International Portfolios", *Journal of Finance*, 48, p.1865-1886.

Glosten, L.R., and Jagannathan, R., 1994, "A Contingent Claim Approach to Performance Evaluation", *Journal of Empirical Finance*, 1, p.133-160.

Gouriéroux, Ch., Holly, A., and Montfort, A., 1982, "Likelihood Ratio Test, Wald Test and Kuhn-Tucker Test in Linear Models with Inequality Constraints on the Regression Parameters", *Econometrica*, 50, p.63-80.

Grinblatt, M., and Titman, S., 1989, "Portfolio Performance Evaluation: Old Issues and New Insights, *Review of Financial Studies*, 2, p.393-421.

Hakkio, C.S., and Leiderman L., 1986, "Intertemporal Asset Pricing and the Term Structure of Exchange Rates and Interest Rates", *European Economic Review*, 30, p.324-344.

Hall, A., and Knez, P.J., 1995, "Testing Portfolio Performance", Working Paper, North Caoline State Universtiy.

Hansen, L.P., Heaton, J., and Luttmer, E.G.J.,1995, "Econometric Evaluation of Asset Pricing Models", *Review of Financial Studies*, 8, p.237-274.

Hansen, L.P., and Jagannathan, R., 1991, "Implications of Security Market Data for Models of Dynamic Economies", *Journal of Political Economy*, 99, p.225-262.

Hansen, L.P., and Jagannathan, R., 1997, "Assessing Specification Errors in Stochastic Discount Factor Models", *Journal of Finance*, 52, p.557-590.

Harvey, C.R., 1995, "Predictable Risk and Returns in Emerging Markets", *Review of Financial Studies*, 8, p.773-816.

Hicks, J.R., 1939, *Value and Capital*, Oxford University Press, Cambridge.

Hirshleifer, D., 1988a, "Residual Risk, Trading Costs, and Commodity Futures Risk Premia", *Review of Financial Studies*,1, p.173-193.

Hirshleifer, D., 1988b, "Risk, Futures Pricing, and the Organization of Production in Commodity Markets", *Journal of Political Economy*, 96, p.1206-1220.

Hirshleifer, D., 1989, "Determinants of Hedging and Risk Premia in Commodity Futures Markets", *Journal of Financial and Quantitative Analysis*, 24, p.313-331.

Hirshleifer, D., 1990, "Hedging Pressure and Futures Price Movements in a General Equilibrium Model", *Econometrica*, 58, p.411-428.

Huberman, G., and Kandel, S., 1987, "Mean Variance Spanning", *Journal of Finance*, 42, p.873-888.

Huberman, G., Kandel, S., and Stambaugh, R.F., 1987, "Mimicking Portfolios and Exact Arbitrage Pricing", *Journal of Finance*, 43, p.1-9.

Huberman, G., and Kandel, S., 1987, "Mean- Variance Spanning", *Journal of Finance*, 42, p.873-888.

Hull, J.C., 1997, *Options, Futures, and Other Derivatives*, Prentice Hall International, London.

Ingersoll, J.E., 1987, *Theory of Financial Decision Making*, Rowman & Littlefield, Maryland.

Jagannathan, R., 1985, "An Investigation of Commodity Futures Prices Using the Consumption-based Capital Asset Pricing Model", *Journal of Finance*, 40, p.175-191.

Jobson, J.D., and Korkie, B., 1982, "Potential Performance and Tests of Portfolio Efficiency", *Journal of Financial Economics*, 10, p.433-466.

Jobson, J.D., and Korkie, B., 1984, "On the Jensen Measure and Marginal Improvements in Portfolio Performance: A Note", *Journal of Finance*, 39, p.245-251.

Jobson, J.D., and Korkie, B., 1989, "A Performance Interpretation of Multivariate Tests of Intersection, Spanning and Asset Pricing", *Journal of Financial and Quantitative Analysis*, 24, p.185-204.

Kandel, S.., and Stambaugh, R.F., 1987, "On Correlations and Inferences about Mean-Variance Efficiency", *Journal of Financial Economics*, 18, p.61-90.

Kandel, S., and Stambaugh, R.F., 1989, "A Mean-Variance Framework for Tests of Asset Pricing Models", *Review of Financial Studies*, 2, p.125-156.

Keynes, J.M., 1930, *A Treatise on Money, Vol. II*, McMillan, London.

Kodde, D.A., and Palm, F.C., 1986, "Wald Criteria for Jointly Testing Equality and Inequality Restrictions", *Econometrica*, 54, p.1243-1248.

Luttmer, E.G.J., 1996, "Asset Pricing in Economies with Frictions", *Econometrica*, 64, p.1429-1467.

Markowitz, H.M., 1952, "Portfolio Selection", *Journal of Finance*, 7, p.77-91.

Markowitz, H.M., 1991, *Portfolio Selection*, Basil Blackwell, Cambridge, Massachusetts.

McCurdy, Th.H., and Morgan, I.G., 1987, "Tests of the Martingale Hypothesis for Foreign Currency Futures with Time-Varying Volatility", *International Journal of Forecasting*, 3, p.131-148.

Newey, W.K., and West, K.D., 1987a, "A Simple Positive Semi-definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix", *Econometrica*, 55, p.703-708.

Newey, W.K., and West, K.D., 1987b, "Hypothesis Testing with Efficient Methods of Moments Estimation", *International Economic Review*, 28, p.777-787.

Pagan, A.R., 1984, "Econometric Issues in the Analysis of Regressions with Generated Regressors", *International Economic Review*, 25, p.221-247.

Peresetsky, A., and DeRoon, F.A., 1997, "Risk Premia in the Ruble/Dollar Futures Market", *Journal of Futures Markets*, 17, p.191-214.

Ross, S.A., 1978, "Mutual Fund Separation in Financial Theory: The Separating Distributions", *Journal of Economic Theory*, 17, p.254-286.

Schwartz, E.S., 1997, "The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging", *Journal of Finance*, 52, p.923-974.

Shanken, J., 1987, "Multivariate Proxies and Asset Pricing Relations: Living with the Roll Critique", *Journal of Financial Economics*, 18, p.91-110

Shanken, J., 1990, "Intertemporal Asset Pricing: An Empirical Investigation", *Journal of Econometrics*, 45, p.99-120.

Snow, K.N., 1991, "Diagnosing Asset Pricing Models Using the Distribution of Asset Returns", *Journal of Finance*, 46, p.955-983.

Solnik, B., 1991, *International Investments* Addison Wesley.

Stoll, H.R., 1979, "Commodity Futures and Spot Price Determination and Hedging in Capital Market Equilibrium", *Journal of Financial and Quantitative Analysis*, 14, p.873-895.

Treynor, J., and Mazuy, F., 1966, "Can Mutual Funds Outguess the Market?", *Harvard Business Review*, 44, p.131-136.

Vasicek, O., 1977, "An Equilibrium Characterization of the Term Structure", *Journal of Financial Economics*, 5, p.177-188.

Samenvatting (Summary in Dutch)

Twee belangrijke problemen in de financiële economie zijn de portefeuillekeuze van beleggers en de prijsvorming van financiële activa. Deze twee problemen staan centraal in deze studie. Zoals de titel reeds suggereert, bestaat de studie uit twee delen. Het eerste deel, dat zich richt op *spanning* en *intersection* richt zich met name op de portefeuillekeuze van beleggers. In het tweede deel, dat handelt over risicopremies op futures markten, wordt een analyse gegeven van de prijsvorming van een specifieke categorie van financiële activa: futures contracten.

De centrale vraag in deel I van deze studie is of beleggers met een bestaande beleggingsportefeuille de risico- en rendementskarakteristieken van deze portefeuille kunnen verbeteren door nieuwe beleggingsobjecten aan hun portefeuille toe te voegen. In termen van de welbekende efficiënte grenslijn wordt gesproken van *intersection* wanneer de efficiënte grenslijn van de initiële beleggingsobjecten en de efficiënte grenslijn van de initiële en de nieuwe beleggingsobject samen, precies één punt met elkaar gemeen hebben. Wanneer deze twee efficiënte grenslijnen geheel met elkaar samenvallen dan wordt gesproken van *spanning*.

In hoofdstuk 2 wordt een inleiding op mean-variance spanning en intersection gegeven en worden deze begrippen gerelateerd aan andere delen van de financieel-economische literatuur. Omdat spanning en intersection niet alleen geïnterpreteerd kunnen worden in termen van efficiënte grenslijnen, maar ook in termen van de zogenaamde *volatility bounds* (volatiltiteitsgrenslijnen) voor prijskernen (*pricing kernels*) zoals geïntroduceerd door Hansen & Jagannathan (1991), wordt allereerst ingegaan op de dualiteit tussen efficiënte grenslijnen en volatility bounds. Aangetoond wordt dat, met uitzondering van de algehele minimum-variantie portefeuille, iedere portefeuille op de efficiënte grenslijn correspondeert met een uniek punt op de volatility bound. De implicaties van mean-variance spanning en intersection voor de rendementsverdeling van financiële activa kunnen worden afgeleid uitgaande van efficiënte grenslijnen of uitgaande van volatility bounds. Beide grenslijnen impliceren identieke restricties voor de rendementsverdeling van financiële activa.

Statistische toetsen voor spanning en intersection kunnen geïnterpreteerd worden met behulp van prestatiemaatstaven zoals de Jensen maatstaf en de Sharpe ratio. De Jensen maatstaf van een nieuw beleggingsobject gemeten ten opzichte van de beleggingsobjecten die de belegger reeds in portefeuille heeft, kan gebruikt worden om de maximale verbetering in de Sharpe ratio te bepalen die resulteert wanneer het nieuwe beleggingsobject aan de portefeuille wordt toegevoegd. De maximale verbetering in de Sharpe ratio's is op zijn beurt bepalend voor de grootte van de spanning-en intersection-toetsen. De regressies en toetsgrootheden die voor toetsen op intersection en spanning worden gebruikt hebben derhalve ook een duidelijke economische interpretatie. Bovendien geven de geschatte regressiecoëfficiënten in combinatie met de initiële beleggingsportefeuille ook aan hoe de nieuwe optimale portefeuille kan worden bepaald. Tenslotte wordt in hoofdstuk 2 aangetoond dat intersection ook gerelateerd is aan de *specification error bounds* (specificatiefoutgrenzen) die onlangs door hansen & Jagannathan (1997) zijn geïntroduceerd. Wanneer de zogenaamde minimum-variantie prijskern voor een deelverzameling van financiële activa als proxy wordt gebruikt om een grotere verzameling van financiële activa te prijzen, dan kan de specification error bound voor deze proxy worden gebruikt als maatstaf voor de afwijking van intersection.

In hoofdstuk 3 worden drie generalisaties gegeven van het in hoofdstuk 2 geïntroduceerde begrip mean-variance spanning. Allereerst laten we in dit hoofdstuk zien hoe regressietechnieken kunnen worden gebruikt om te toetsen op spanning voor meer algemene nutsfuncties. Ten tweede laten we in hoofdstuk 3 zien hoe de toetsen voor spanning gegeneraliseerd kunnen worden naar het geval waarin sommige beleggingsobjecten, zoals futures, forwards en swaps, geen investering vereisen. Tenslotte laten we in hoofdstuk 3 zien hoe in toetsen voor spanning rekening gehouden kan worden met het feit dat sommige activa niet verhandelbaar zijn. Wanneer een belegger een niet-verhandelbare positie in een bepaald activum heeft, veranderen hierdoor zijn totale beleggingsopbrengsten. In de toetsen voor spanning kan hiermee rekening worden gehouden door de rendementen op beleggingsobjecten aan te passen voor het rendement op de niet-verhandelbare positie.

In hoofdstuk 3 wordt vervolgens getoetst of er spanning is van een aantal goederen en valuta futures door een drietal internationale aandelenindices: de S&P 500, de FAZ (Duitsland) en de FTSE (Verenigd Koninkrijk). Wanneer verondersteld wordt dat de relevante momenten van de rendementsverdelingen constant zijn, dan kan de nul-hypothese van spanning voor de meeste futures contracten worden verworpen. Of de spanning-hypothese voor een bepaald futures contract wel of niet verworpen wordt, hangt echter wel af van de specifieke nutsfunctie waarin we zijn geïnteresseerd. Wanneer een belegger een niet-verhandelbare positie heeft in één van de onderliggende waarden van de goederen futures, dan kan de spanning-hypothese bijna altijd worden verworpen, ongeacht de specifieke nutsfunctie. Bovendien impliceert een niet-verhandelbare positie in één van de agrarische goederen (zoals tarwe of maïs) dat de spanning-hypothese voor de meeste agrarische futures contracten wordt verworpen. In tegenstelling tot een niet-verhandelbare positie in één van de goederen, blijkt dat wanneer een belegger een niet-verhandelbare positie in een valuta heeft, de spanning-hypothese alleen wordt verworpen voor beleggers met een zogenaamde *power*-nutsfunctie die een voorkeur voor scheefheid weerspiegelt. Wanneer we rekening houden met het feit dat futures rendementen deels voorspeld kunnen worden met behulp van de posities die grote (hedge-)partijen in futures markten innemen, dan kan de spanning-hypothese worden verworpen voor de meeste futures contracten, ongeacht de specifieke nutsfunctie van de belegger.

In de hoofdstukken 2 en 3 wordt steeds verondersteld dat er geen marktfricties zoals beperkingen ten aanzien van short-sales en transactiekosten bestaan. In hoofdstuk 4 laten we zien hoe regressietechnieken ook gebruikt kunnen worden voor spanning- en intersection-toetsen wanneer zulke short-sales restricties en/of transactiekosten wel bestaan. In hoofdstuk 4 wordt tevens een empirische toepassing van deze spanning-toetsen gegeven voor een aantal opkomende markten (*emerging markets*). Wanneer de gebruikelijke toetsen voor mean-variance spanning, zoals beschreven in hoofdstuk 2, worden gebruikt dan blijkt dat de spanning-hypothese voor de zogenaamde opkomende markten ten opzichte van de MSCI aandelenindices voor de Verenigde Staten, Europa en Japan, eenvoudig verworpen kan worden. Ook wanneer rekening gehouden wordt met het feit dat noch voor de opkomende markten noch voor de MSCI Indices short-selling mogelijk is, wordt de spanning-hypothese voor vele opkomende markten verworpen. De opkomende markten waarvoor de spanning-hypothese wordt verworpen, ook rekening houdend met short sales restricties, blijken echter doorgaans markten te zijn waarvoor aandelenbezit door buitenlandse beleggers slechts in zeer beperkte mate mogelijk is.

Wanneer rekening gehouden wordt met het bestaan van transactiekosten, dan is het empirisch bewijs tegen de spanning-hypothese veel zwakker. Voor beleggers met een beleggingshorizon van twee maanden of langer lijken de opkomende markten echter nog steeds diversificatievoordelen ten opzichte van de drie MSCI Indices te bieden. Voor het geval waarin beleggers maandelijks hun portefeuille verhandelen, wordt de kritische grens van de transactiekosten in iedere opkomende markt geschat waarvoor de spanning-hypothese niet verworpen kan worden. Voor bijna alle opkomende markten blijkt deze kritische grens lager te zijn dan de werkelijke transactiekosten in de betreffende markt.

De empirische analyse in hoofdstuk 4 suggereert derhalve dat wanneer we - uitgaande van een Amerikaanse belegger - de potentiële diversificatievoordelen van opkomende markten ten opzichte van ontwikkelde markten willen vaststellen, we terdege rekening moeten houden met het bestaan van marktfricties zoals short-sales restricties en het bestaan van transactiekosten. Ook wanneer we rekening houden met deze fricties blijken er echter nog steeds diversificatievoordelen voor de opkomende markten te bestaan. Wanneer de analyse echter beperkt wordt tot de periodes na de liberaliseringen die in de meeste opkomende markten hebben plaatsgevonden en wanneer we rekening houden met het feit dat buitenlandse beleggers vaak slechts in beperkte mate aandelen in opkomende markten kunnen kopen, dan resteert er nog zeer weinig bewijs ten gunste van deze diversificatievoordelen.

Deel II van deze studie gaat over de modellering van risicopremies op futures markten. In hoofdstuk 6 wordt een eenvoudig meer-factoren model geanalyseerd waarin de risicopremies op futures markten bepaald worden door de covariantie van het futures rendement met het rendement op de marktportefeuille en door zogenaamde *hedging pressure* variabelen. *Hedging pressure* geeft aan in welke mate hedgers per saldo een short of een long positie in futures contracten hebben. In het meer-factoren model zijn de relevante toestandsvariabelen de hedging pressure van het futures contract zelf alsmede van aanverwante futures contracten. Hedging pressures van andere futures contracten worden aangeduid als *cross* hedging pressures. In hoofdstuk 6 wordt een empirische analyse van het meer-factoren model en van het CAPM gegeven voor een twintigtal futures-contracten die in vier groepen verdeeld kunnen worden: financiële, agrarische, minerale en valuta futures. In de gebruikte specificatie van het meer-factoren model zijn de hedging pressures binnen iedere groep de relevante toestandsvariabelen.

De empirische analyse toont aan dat de hedging pressure variabelen een significant effect op futures rendementen hebben, rekening houdend met markt risico. Deze bevinding is in overeenstemming met het meer-factoren model. In termen van de misspecificatie-maatstaf zoals geïntroduceerd door Hansen & Jagannathan (1997) leidt het model met hedging pressure variabelen tot een significante reductie in misspecificatie ten opzichte van een model zonder hedging pressure variabelen. Fouten in het prijzen van (portefeuilles van) futures contracten zijn derhalve aanzienlijk kleiner wanneer naast marktrisico ook rekening wordt gehouden met hedging pressure variabelen.

In hoofdstuk 7 wordt tenslotte een analyse gegeven van de verschillen in één-periode rendementen voor futures contracten met dezelfde onderliggende waarde, maar met verschillende looptijden. Deze verschillen kunnen worden toegeschreven aan risicopremies in de termijnstructuur van futures *yields*, waarbij de yield gedefinieerd is als het procentuele verschil tussen de futures prijs en de prijs van de onderliggende waarde (op jaarbasis). Deze yield wordt bepaald door variabelen zoals (binnen- en buitenlandse) rentes, dividenden, voorraadkosten, convenience yields, etc.

Onze analyse richt zich op de informatie die de termijnstructuur van futures prijzen (yields) bevat over toekomstige futures yields en de risicopremies in de yields. Wanneer gebruik wordt gemaakt van een eenvoudig één-factor model, waarvan het Vasicek-model en het CIR-model speciale gevallen zijn, dan kunnen precieze uitspraken worden gedaan over de informatie die aanwezig is in de huidige termijnstructuur van futures prijzen. Aangetoond wordt dat de termijnstructuur van futures prijzen alleen in het geval van het Vasicek-model geen informatie over risicopremies bevat omdat deze premies in het Vasicek-model constant zijn.

De empirische analyse in hoofdstuk 7 laat zien dat het Vasicek-model niet verworpen kan worden voor futures op olie en op Duitse marken. Voor futures op goud en sojabonen blijken de verwachte futures rendementen - en daarmee de risicopremies - af te hangen van de helling van de termijnstructuur van futures prijzen. Hierbij blijkt het verwachte rendement op lange termijn contracten ten opzichte van korte termijn contracten kleiner (groter) te zijn, naarmate het verschil tussen futures prijzen voor lange en korte looptijden groter (kleiner) is, d.w.z. naarmate de termijn structuur een steiler (minder steil) verloop kent. Voor futures op slachtvee zijn de empirische bevindingen niet eenduidig. Hoewel voor futures contracten op goud, sojabonen en slachtvee de risicopremies lijken af te hangen van het verloop van de termijnstructuur van futures prijzen, kan de variatie in risicopremies voor deze contracten niet worden verklaard door het CIR-model.

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Frans de Roon graduated in Business Administration (Finance) in 1993. From 1993 till 1996 he was a Ph.D.-student at the Department of Finance and CentER for Economic Research of Tilburg University. Part of his work has been published in the Journal of Financial and Quantitative Analysis and in the Journal of Futures Markets. His joint work on emerging markets has received the second award at the Robeco Portfolio Management Workshop, 1997. His current affiliation is the Department of Finance of the Erasmus University in Rotterdam.

The portfolio choices of investors and asset pricing are two important topics in financial economics. These two topics form the main theme of this study. The first part of the study, which is about spanning and intersection, mainly focuses on the portfolio choices of investors. Building on the well known mean-variance portfolio theory of Markowitz, we analyze whether investors can extend their efficient set by including additional securities in their portfolio, which comes down to evaluating the performance of the additional assets. The analysis of this portfolio question is extended to the case where investors have non mean-variance utility functions, where investors face nonmarketable risks, and where investors face short sales constraints and transaction costs. Empirical applications for the analysis in the first part are given for futures markets and for emerging markets. The second part of this study is about risk premia in futures markets. In this part, we first provide an empirical analysis of the effects that the presence of hedgers has on futures risk premia. This effect is known as the so called hedging pressure. Finally, we give an empirical analysis of the differences in risk premia for futures contracts that differ in their maturity only.

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